



# Numerical Analysis of Cu–H<sub>2</sub>O Nano-Fluid Natural Convection in a Trapezoidal Enclosure Filled Withporous Medium

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## Abstract

The present simulation focused on natural convection in a semi-trapezoidal porous enclosure filled with water-based nanofluid. The mathematical model has been formulated in dimensionless stream function and temperature taking into account the Darcy–Boussinesq approximation. The Tiwari and Das’s nano fluid model with new more realistic empirical correlations for the physical properties of the nanofluids has been used for numerical analysis. The developed partial differential equations of the present computational domain are employed by using the vorticity stream function approach and the second ordered approximation finite difference scheme. The house-computational numerical algorithm has been validated against the previous work, and the computational results have been registered with good correlation. The impact of a wide range of governing parameters on fluid flow patterns and temperature gradient variations within the enclosure is exhibited in the form of flow patterns (i.e., streamlines), temperature contours (i.e., isotherms), and the local Nusselt number of the hot wall. The heat distribution from the hot wall is a raising function of buoyancy number-Ra where as an insignificance heat distribution (local Nusselt number) is observed for the impact of porosity parameter and nanofluid volume fraction.

**Keywords** Natural convection · Laminar flow · Porous medium · Nanofluid · Semi-trapezoidal cavity

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## List of Symbols

$C_p$	Specific heat at fixed pressure
$g$	Gravitation acceleration vector
$K$	Porous medium permeability
$k$	Thermal conductivity
$P$	Pressure
$Ra$	Rayleigh number
$T_c$	Cold right wall temperature
$T_h$	Hot left wall temperature
$T$	Temperature
$(u, v)$	Cartesian velocity components
$(U, V)$	Dimensionless velocity components
$V$	Darcian velocity vector
$(x, y)$	Cartesian co-ordinates
$(X, Y)$	Dimensionless co-ordinates

## Greek letters

$\beta$	Thermal expansion coefficient.
$\phi$	Uniform concentration of the nanoparticles
$\psi$	Stream function
$\mu$	Dynamic viscosity
$\rho$	Density
$\varepsilon$	Porosity of the porous medium
$\theta$	Non-dimensional temperature

## Subscripts

$c$	Cold
$f$	Fluid
$h$	Hot
$mnf$	Nanofluid saturated porous medium
$m$	Clear fluid saturated porous medium
$nf$	Nano fluid
$s$	Particle

## Introduction

Different experimental and numerical investigations have been conducted on the convective heat transfer due to buoyancy in cavities is a major topic. In addition to its numerous practical and industrial applications, its interest is justified by the fact that it is highly versatile, including energy systems, medicine, geosciences and chemical process engineering, heat transfer enhancement in petroleum reservoirs, etc. The natural convection induced by temperature differences in walls has been the subject of many studies in recent decades, bearing

in mind different forms of the cavities such as rectangular [1–3], square [4–7] triangular [8–12] trapezoidal [13–15].

Using the buoyancy force to model thermal fields in complex geometry is quite challenging. Thus, researchers have carried out significant studies on natural convection in the non-rectangular enclosures during the last two decades. Below are a few examples of natural convection studies conducted in non-rectangular enclosures. Dutta et al. [16] analyzed entropy based MHD convection, and heat distribution in a Rhombus shaped container packed with cu-water nanofluids. The rate of entropy generation diminished with the growth of magnetic parameter, Rayleigh number and inclination Angles of the cavity. Shayan et al. [17] considered a L-shaped cavity with various baffle configurations filled Cu-water nanofluid for the heat transport and fluid motion examination by LBM approach. Enhancement of natural convection irrespective of baffle positions observed for the large values of Rayleigh numbers and length of Baffle also shown an impact in this case. Masoud et al. [18] also considered a different L-shaped enclosure filled with Al<sub>2</sub>O<sub>3</sub>-H<sub>2</sub>O nanomaterial for the elaborated analysis of free convective heat transport using entropy optimization. Based on cost of the nanofluid first time they introduced economic analysis for the assessment of performance of the cavity and got good agreement with the existed results. Decrement of entropy optimize number observe for the rising values of nanoparticle concentration. Raizah et al. [19] carried out three distinct thermal conditions like conducting, cold and hot solid particles within the E-shaped enclosure partially filled with porous medium to assess the flow behavior and thermal distribution of nanofluid. From the results it is revealed that the fluid motion and thermal distribution is strengthened within the enclosure for the condition of hot solid particles. Keramata et al. [20] took a H-shaped enclosure packed with aluminum water nanofluid by installing a baffle in the enclosure for the analysis of flow and heat expansion of nanofluid. They maintained hot temperature at the top rib, low temperature at side walls and other walls of cavity are insulated. Rise of heat transmission observed for the enhancement of Buoyancy parameter (i.e., Ra—Rayleigh number) and nano-particle volume fraction. Liu et al. [21] also studied with entropy generation, convective flow momentum of water—(Al<sub>2</sub>O<sub>3</sub>) aluminum trioxide nanofluid inside the enclosure formed by connecting two inclined triangular cavities with the implementation of horizontal magnetic field. In this geometry bottom portion of the right wall of the computational domain maintained Hot and rest of the walls kept adiabatic. As a result, decrement in Entropy generation and increment in Bejan number observed for high inclination angle.

A great deal of research has been conducted on free convection in porous media, one of the most widely used technologies in many different fields. For example, food processing, porous bearing, solar collectors, water movement in geothermal reservoirs, heat dissipation from chips via porous metal foams, underground spreading of chemical wastes, thermal insulation, evaporative cooling, and solidification and design of porous insulation for a nuclear power reactor core are the main applications. Roy et al. [22] numerically investigated the natural convection in the concentric annulus filled with non-Darcy porous media along with a chemically reacting fluid. We consider the non-Darcy model to elucidate the problem that includes the afore said effects. Later, Saha et al. [23] reported natural convection in the wavy enclosure filled with non-Darcy porous media along with a chemically reacting fluid. They are found opposite characteristics when an increase in the Forchheimerdrag parameter. Sivasankaran et al. [24] adopted non-Darcy porous model for the analysis of convective flow and heat expansion of Casson liquid within a porous square enclosure with sinusoidal thermal radiation. By SIMPLE algorithm, results states that greater Casson parameter values under radiation generates thermal stratification phenomenon. Deterioration of heat transport for the rising values of radiative flux  $R_d$  parameter and improvement of heat transfer noticed for

the increment of Casson parameter. Alsabery et al. [25] also considered trapezoidal cavity for the analysis of Darcian free convection in some portion of enclosure is embedded with porous medium and rest of the portion is filled with nanofluid. They filled the cavity with water-based nanofluid and the nanoparticles Ag or TiO<sub>2</sub> or Cu chose for the analysis. As a result, convection is remarkably increased with the addition of Ag-water nanofluid.

Adding solid nanoparticles to a base fluid increases thermal conductivity, which is an effective method to increase heat transfer. Nanoparticles in a liquid increase the thermal conductivity of the mixture, called a nanofluid, owing to the thermo-physical properties of nanoparticles. Due to this property, nanofluids can achieve a significant increase in heat transfer without consuming a large amount of energy. Roy et al. [26] have performed finite difference method to study natural convection of nanofluids in a square enclosure with different shapes of inner geometry. They observed that the Nusselt number at the inner and outer cylinders diminishes with circular, elliptical, and rectangular inner shapes. Parvin et al. [27] conducted a numerical study on Heat transfer characteristics of nanofluids from a sinusoidal corrugated cylinder placed in a square cavity. They stated that the concentration of nanoparticles increases, the average Nusselt number at the internal and external cylinders becomes stronger. Further, Dusty nano fluid flow within a concentric annulus was studied numerically by Saha et al. [28] to investigate the temperature profiles and heat transfer. He described that the temperature profiles of the dusty flow varied from the respective pure fluid flow and heat transfer was significantly affected by the parameters involved with the dusty particles. Chandra Roy et al. [29] studied Flow and heat transfer characteristics of a nanofluid between a square enclosure and a wavy wall obstacle with vertical isothermal coldwalls and bottom adiabatic walls. Results showed that the maximum and minimum values of the velocity profile increase with the higher Rayleigh number.

Evidently, no study has taken as an attempt to investigate the natural convection in the semi-trapezoidal enclosure filled with porous matrix. In the present study, our aim is to simulate the natural convection of nano fluid flow (Tiwari and Das's model) in a semi-trapezoidal enclosure embedded with porous medium with pressure elimination and finite difference approach. Darcian model is imposed to govern the model equations. The 2D computational domain and the corresponding governing partial differential equations are employed through stream function approach. The impact of thermal Rayleigh number ( $Ra$ ), nanofluid volume fraction ( $\phi$ ) and Porosity parameter( $\epsilon$ ) on streamline and isotherm distributions are presented via isotherms and streamline patterns. To the author's knowledge, a natural convection in a semi-trapezoidal enclosed space filled with a porous medium has not yet been investigated. This gap is therefore being filled by the present research.

## Mathematical Modelling

The 2-D computational domain under study comprises unsteady laminar convective flow of water-based nanofluid within an enclosure (trapezoidal enclosure) containing a Darcian porous medium. The porous material is considered to be isotropic, homogeneous and filled with an incompressible Cu-Water nano fluid. The fluid and solid matrix are at the same temperature, i.e., local thermal equilibrium. A schematic diagram of the domain of interest in an ( $x$ ,  $y$ ) coordinate system with boundaries is presented in Fig. 1. The thermal boundary conditions are imposed. The left inclined cold isothermal wall is sustained at temperature TC (cold wall), the right vertical wall at TH (hot wall), and the upper and base walls are adiabatic. It is assumed that nanoparticles are suspended in the nanofluid using either surfactant or

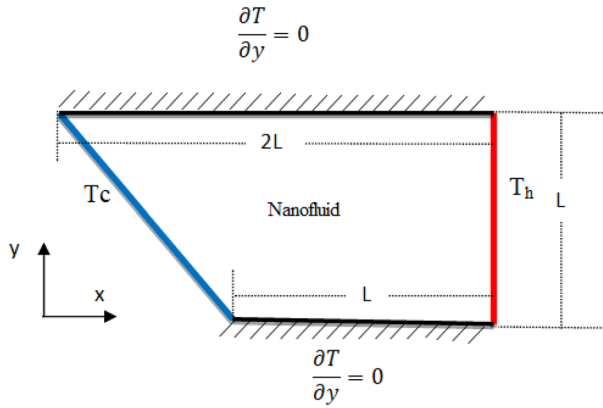


Fig. 1 Schematic diagram of the domain

surface charge technology. This prevents nanoparticles from agglomeration and deposition on the porous matrix. The fluid motion is due to buoyancy force owing to density contrast due to temperature variations with gravity  $g$  is acting downward. Chemical reaction, thermal dispersion, thermal stratification effects are also ignored. The proposed model is governed by mass, momentum, and energy conservations. So, under Boussinesq approximation, these equations can be written as (see [22, 23, 30]):

$$\nabla \cdot V = 0 \tag{1}$$

$$\nabla p = -\frac{\mu_{mf}}{K} V - (\rho\beta)_{mf}(T - T_c)g \tag{2}$$

$$(V \cdot \nabla)T = \frac{k_{mf}}{(\rho C_p)_{mf}} (\nabla^2 T) \tag{3}$$

The nanofluid physical properties: thermal conductivity ( $k_{mf}$ ), viscosity ( $\mu_{mf}$ ), heat capacitance ( $\rho C_p)_{mf}$  and due to gravity with buoyancy coefficient  $(\rho\beta)_{mf}$ .

$$\left. \begin{aligned} \mu_{mf} &= \frac{\mu_f}{(1 - \phi)^{2.5}} \\ (\rho C_p)_{mf} &= (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s \\ (\rho\beta)_{mf} &= (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s \\ \frac{k_{mf}}{k_f} &= \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \end{aligned} \right\} \tag{4}$$

Here indices ‘nf’, ‘f’ and ‘s’ indicates nanofluid, pure fluid and nanoparticle and  $\phi$  is referred as nanoparticles concentration. The nanofluid dynamic viscosity ( $\mu_{mf}$ ) can be expressed in terms of base fluid viscosity ( $\mu_f$ ) consisting of the fine spherical shapes particles stated by [31]. In Table 1 explored the nanofluid thermophysical properties. These thermophysical properties are also used by Khanafer et al. [32], Oztop [33], Abu-Nada [34], and Muthamilselvan et al. [35].

Further it should be noticed that for  $k_s \gg k_f$  it limitation as follows.

$$k_{mf} \approx k_f \frac{1 + 2\phi}{1 - \phi} \tag{5}$$

**Table 1** Thermal-physical properties of water and Cooper (Cu) nanoparticles (see [42])

Physical properties	Base fluid (water)	Cu
$C_p(J.kg^{-1}.K^{-1})$	4179	385
$\rho(kg.m^{-3})$	997.1	2700
$k(W.m^{-1}.K^{-1})$	0.613	205
$\alpha \times 10^{-7}(m^2.s^{-1})$	1.47	846.4
$\beta \times 10^{-5}(K^{-1})$	21	2.22

Alter, from the study of Bejan [36]

$$(\rho C_p)_m = \varepsilon(\rho C_p)_f + (1 - \varepsilon)(\rho C_p)_s, k_m = \varepsilon k_f + (1 - \varepsilon)k_s \tag{6}$$

By the expressions Eqs. (6) and (4), nanofluid physical properties along the Das model in a porous trapezoidal enclosure are expressed by

$$\left. \begin{aligned} (\rho C_p)_{mnf} &= \varepsilon(\rho C_p)_{nf} + (1 - \varepsilon)(\rho C_p)_s = (\rho C_p)_m \left[ 1 - \varepsilon\phi \frac{(\rho C_p)_f - (\rho C_p)_s}{(\rho C_p)_m} \right] \\ k_{mnf} &= \varepsilon k_{nf} + (1 - \varepsilon)k_s = k_m \left\{ 1 - \frac{3\varepsilon\phi k_f(k_f - k_s)}{k_m[k_s + 2k_f + \phi(k_f - k_s)]} \right\} \\ \alpha_{mnf} &= \frac{k_{mnf}}{(\rho C_p)_{nf}} \end{aligned} \right\} \tag{7}$$

Subscripts ‘‘mnf’’, ‘‘s’’, and ‘‘m’’ are referring tonano liquid, solid, and clear fluids. The thermal conductivity enhancement adopted with Eq. (7), it is well worked in the work of Yu et al. [37].

The Cartesian form of the Eqs. (1), (2), (3) can be expressed as follows (see [38, 39, 43]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{8}$$

$$\frac{\mu_{mnf}}{K} \left( \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right) = -g(\rho\beta)_{nf} \frac{\partial T}{\partial x} \tag{9}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{mnf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{10}$$

$$X = \frac{x}{L}, Y = \frac{y}{L}, U = uL/\alpha_{mnf}, V = vL/\alpha_{mnf}, \theta = (T - T_c)/(T_h - T_c) \tag{11}$$

The fluid flow field introduced stream function  $\psi$  and it is defined as follows

$$U = \frac{\partial \psi}{\partial y}, V = -\frac{\partial \psi}{\partial x} \tag{12}$$

so, the governing equations are taken with account of stream function as follows

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -Ra.H(\varphi) \frac{\partial \theta}{\partial x} \tag{13}$$

$$\frac{\partial \psi}{\partial Y} \frac{\partial \theta}{\partial X} - \frac{\partial \psi}{\partial X} \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} \tag{14}$$

The dimensionless boundary conditions of domain of interest are

$$\left. \begin{aligned} \psi = 0, \theta = 0 \text{ on slant wall} \\ \psi = 0, \theta = 1 \text{ on right wall} \\ \psi = 0, \frac{\partial \theta}{\partial Y} = 0 \text{ on } Y = 0 \text{ and } Y = 1 \end{aligned} \right\} \tag{15}$$

Here  $Ra = gK(\rho\beta)_f(T_h - T_c)L/(\alpha_m\mu_f)$ .

$$\text{and } H(\varphi) = \frac{[1-\varphi+\varphi(\rho\beta)_p/(\rho\beta)_f][1-\varphi+\varphi(\rho C_p)_p/(\rho C_p)_f]}{1-\frac{3\varepsilon\varphi k_f(k_f-k_p)}{k_m[k_p+2k_f+\varphi(k_f-k_p)]}}(1-\varphi)^{2.5}$$

The physical quantities of interest are the local Nusselt number  $Nu$ , which is defined as

$$Nu = -\frac{k_{mnf}}{k_f} \left( \frac{\partial \theta}{\partial X} \right)_{X=1} \tag{16}$$

and the average Nusselt number  $Nu_{avg}$  is given by

$$Nu_{avg} = \int_0^1 Nu dY \tag{17}$$

### Numerical Method

The stream function Poisson Eq. (14) is employed from the gauss–seidel iterative method. After that the modelling of partial coupled vorticity transport differential Eq. (13) of the present investigation of the domain and the corresponding boundary conditions of the semi-trapezoidal boundaries (15) are solved with finite difference with second order accuracy. The developed computation algorithm is ended when the Poisson’s equation of stream function residual reaches below  $10^{-8}$ . The house-computational code is developed in MATLAB and verified converges against the work of [40] and [41]. Table 2 describes the comparison of average Nusselt number for different Rayleigh numbers  $Ra$  and the solid volume fraction for other authors and it gives a good agreement.

For the grid independence study, working substance has been taken as Cu – water nano fluid and the simulation was run with  $\phi = 0.05$ ,  $Ra = 500$  and  $\varepsilon = 0.05$ . Grid independence test was carried out to determine the optimum grid size for the present study. Six different grid sizes ( $80 \times 40$ ,  $100 \times 50$ ,  $120 \times 60$ ,  $140 \times 70$ ,  $160 \times 80$ , and  $180 \times 90$ ) were tested

**Table 2** Comparison of the average Nusselt number in a porous triangular enclosure

Ra	$\varphi$	Nu <sub>avg</sub>		
		Ref. [37]	Ref. [38]	Present
500	0	9 .66	9 .52	9.63
1000	0	13.9	13.6	13.96
500	0.1	9.42	9.44	9.43

**Table 3** Grid independence with  $\phi = 0.05$ ,  $Ra = 500$  and  $\varepsilon = 0.05$ 

Grid size	80 × 40	100 × 50	120 × 60	140 × 70	160 × 80	180 × 90
Nu_avg	7.9295	7.9721	7.9841	7.9651	7.9670	7.9679

to find out the effect on the average Nusselt number calculated at right hot wall. It has been found that there is no significant change in average Nusselt number beyond the grid size of  $100 \times 50$ , which is shown in Table. 3. Therefore, for the present study, the grid size of  $160 \times 80$  has been used to perform all the simulations.

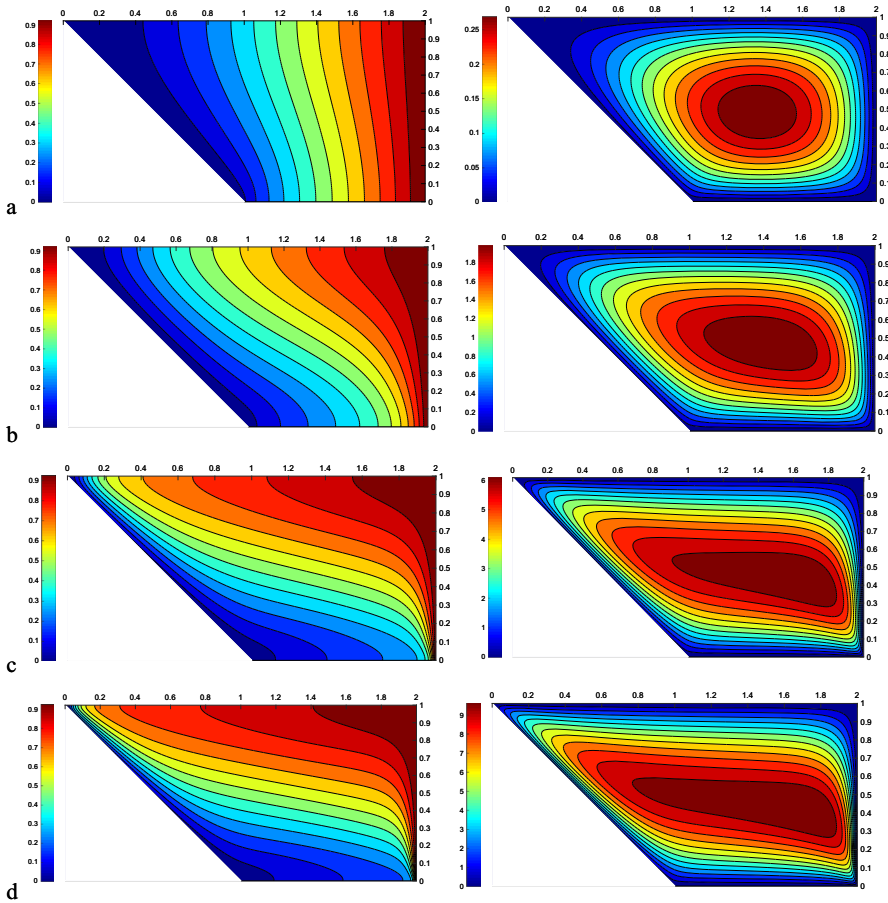
## Results and Discussion

The controlling key parameters have been numerically investigated for the boundary value problems Eqs. (13), (14), (15): The buoyancy number ( $Ra = 1000, 500, 100, 10$ ), the porosity parameter ( $\varepsilon = 0.05, 0.5, 0.8, 1.2$ ), and the solid volume fraction parameter of nanoparticles ( $\phi = 0.04, 0.05$ ). A considerable particular outcome on the impacts of these regulating factors on both the flow field and the thermal distribution of the nanofluid have been noticed.

Dissimilar values of the governing parameter Rayleigh number ( $Ra$ ) cause a substantial change in heat distribution in a trapezoidal regime filled with saturated porous media. Figure 2 displays fluid flow circulations and temperature distributions for water-based Cu–nanofluid for different  $Ra$  values with  $\phi = 0.05$ ,  $\varepsilon = 0.5$  for the porous solid system. Regardless of the Rayleigh number and the nature of the porous media, an anti-clockwise mono circulation is formed inside the computational domain. When  $Ra = 10$ , a single circular eddy is formed inside the enclosure, and the accompanying isotherms occupy the enclosure (see Fig. 2a). When  $Ra$  values grow, the intensity of viscous forces decreases, causing buoyancy forces to increase, and vortex strength inside the cavity to increase. As a result, for  $Ra = 100, 500$ , and  $1000$  (Fig. 2b, c, and d), the corresponding eddies inside the cavity are gradually strengthened, and the eddy form shifts from circular to elliptical. The vortex reaches its maximum intensity at  $Ra = 1000$  (Fig. 2d). When the Rayleigh number increases, the convective fluid cell becomes stronger, the convective circulation extends towards the horizontal axis, and the thermal boundary layer along the cold wall becomes stronger. The former is supported by the maximum absolute values of the stream function, which are as follows:

$$|\psi|_{\max}^{Ra=10} = 0.29 < |\psi|_{\max}^{Ra=100} = 2.16 < |\psi|_{\max}^{Ra=500} = 6.75 < |\psi|_{\max}^{Ra=1000} = 10.78$$

It is evident that changes in free convective fluxes are proportional to changes in the temperature field. At  $Ra = 10$  (Fig. 2a), a mild heat transition from the right hot wall to the left inclined cold wall is noticed, as well as a significant dispersion of isotherms towards the cold wall. The increasing intensity of heat transition along the inclined cold wall is caused by an increase in buoyancy number ( $Ra$ ). The flow particles are enhanced towards the cold wall at  $Ra = 500$  (Fig. 2d), and the thermal contours are steeper and more concentrated towards the cold wall. For higher amounts of buoyancy forces, the development of a thermal flume is observed. The thermal flume is stronger at the thermal Rayleigh number  $Ra = 500$ , as seen in Fig. 2d. The main essential characteristic of the Porosity factor that influences heat expansion inside the cavity. Figure 3 displays flow patterns and isotherms for different values of the Porosity parameter  $\varepsilon$  at  $\phi = 0.05$ ,  $Ra = 500$  for the porous solid system. Increasing values of  $\varepsilon$  result in negligible changes in streamlines and isotherms. Maximum absolute values of



**Fig. 2** Isotherms and streamlines for  $\phi = 0.05, \epsilon = 0.5$ : **a**  $Ra = 10$ , **b**  $Ra = 100$ , **c**  $Ra = 500$ , **d**  $Ra = 1000$

the stream function corroborate this tendency.  $|\psi|_{\max}^{\epsilon=0.8} = 10.7789 < |\psi|_{\max}^{\epsilon=0.5} = 10.7840 < |\psi|_{\max}^{\epsilon=0.05} = 10.7856$ .

Another important parameter that helps us to understand how nanoparticles influence heat transition of nanofluids is the volume percentage of solid parameter ( $\phi$ ). Figure 4 displays flow patterns and temperature curves for various values of, at  $\phi = 0.05, Ra=500$  for the porous solid system. When values are raised, there is a reduction in fluid flow inside the cavity but no effect on thermal expansion. When the concentration of nanoparticles inside the cavity increases, the fluid flows upward along the right heated wall and downward along the left inclined cold wall. Such behaviour can be observed by values of the stream function  $|\psi|_{\max}^{\phi=0.04} = 11.00 > |\psi|_{\max}^{\phi=0.05} = 10.78$  Figs. 5 and 6 show the effect of thermal  $Ra$ , porosity, and nanofluid volume fraction on the local Nusselt number. Heat transmission along the hot wall rises with Rayleigh number; moreover, the influence of Porosity and nanofluid volume fraction on heat dispersion along the hot wall is minimal (Fig. 6).

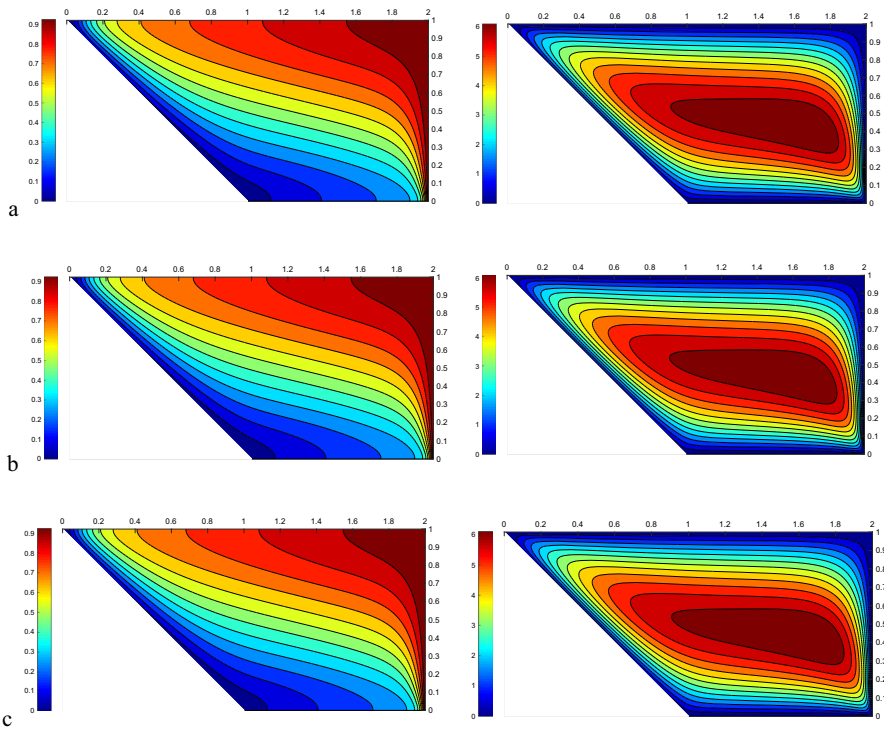


Fig. 3 Isotherms and streamlines for  $\phi = 0.05, Ra = 500$ : **a**  $\varepsilon = 0.05$ , **b**  $\varepsilon = 0.5$ , **c**  $\varepsilon = 0.8$

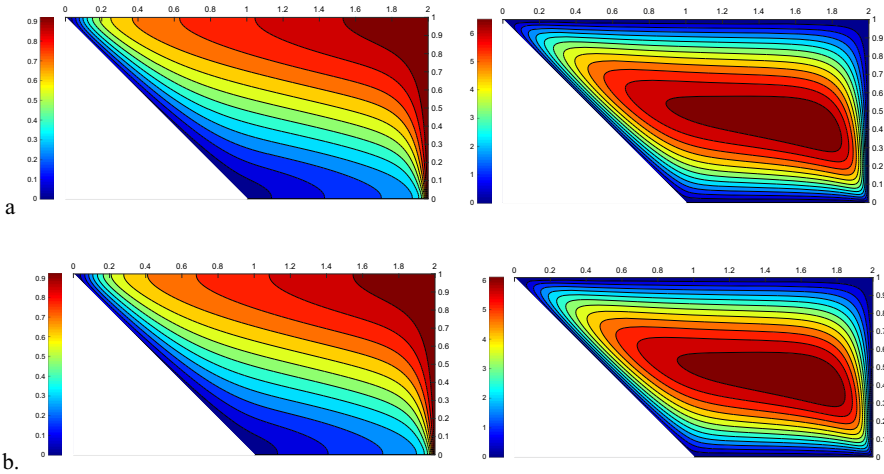


Fig. 4 Isotherms and streamlines for  $\varepsilon = 0.05, Ra = 500$ : **a**  $\phi = 0.02$ , **b**  $\phi = 0.05$

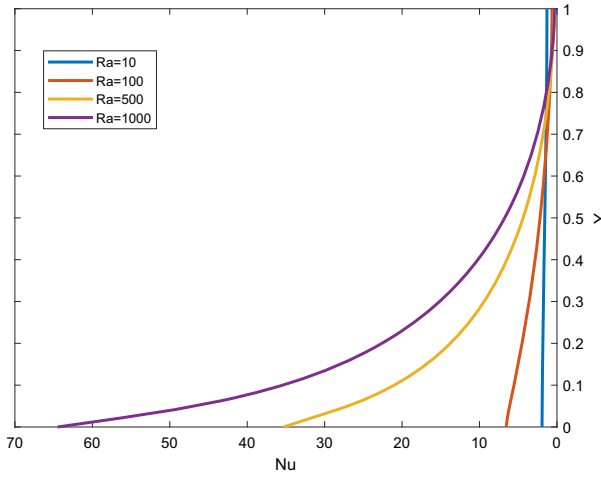


Fig. 5 Local Nusselt number variation of the hot wall with Ra for  $\phi = 0.05, \epsilon = 0.5$

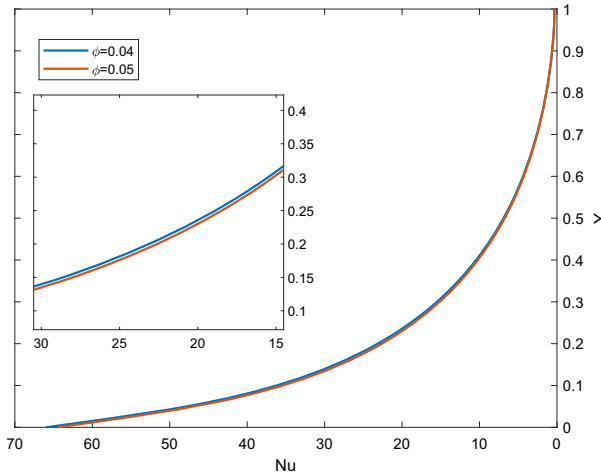


Fig. 6 Local Nusselt number variation of the hot wall with  $\phi$  for  $\epsilon = 0.05, Ra = 1000$

### Conclusions

The pressure elimination based numerical methodology (finite difference method-Vorticity stream function approach) is used to investigate a Copper colloidal solution based nanofluid of natural convective motion in a semi-trapezoidal enclosure embedded with porous media. For Tiwari and Das' nanofluid model, the Darcy-Brinkman model was examined. The effect of buoyancy number (Ra), volume percentage of nanofluid ( $\phi$ ) and Porosity parameter ( $\epsilon$ ) on convective flow and heat distributions is investigated. Based on the numerical analysis using finite differences, it can be concluded that the thermal Rayleigh number (Ra) can be a critical control parameter of thermal distribution and fluid flow momentum. Furthermore,

the heat distribution from the hot wall is a rising function of buoyancy number-Ra, but the influence of porosity parameter and nanofluid volume fraction results in an insignificant heat distribution (local Nusselt number).

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## Declarations

**Conflict of interest** The authors declare that they have no conflict of interest.

## References

1. Nogueira, R.M., Martins, M.A., Ampessan, F.: Natural convection in rectangular cavities with different aspect ratios. *Revista de EngenhariaTérmica* **10**(1–2), 44–49 (2011)
2. Gawas, A.S., Patil, D.V.: Natural convection heat transfer with anisotropic thermal diffusion for tilted two-dimensional cavities. *Int. J. Heat Mass Transf.* **194**, 123000 (2022)
3. Sun, Y., Lin, G., Yu, J., Wen, D., Bai, L.: Theoretical investigation of natural convection heat transfer in inclined and fully divided CO<sub>2</sub> enclosures on Mars. *Int. J. Heat Mass Transf.* **126**, 1113–1122 (2018)
4. Wang, Z.H., Huang, Z., Zhang, W., Xi, G.: A meshless local radial basis function method for two-dimensional incompressible Navier-Stokes equations. *Numer. Heat Transf. Part B Fundam.* **67**(4), 320–337 (2015)
5. Choi, S.K., Kim, S.O.: Comparative analysis of thermal models in the lattice boltzmann method for the simulation of natural convection in a square cavity. *Numer. Heat Transf. Part B Fundam.* **60**(2), 135–145 (2011)
6. Benchabi, R., Lanani, A.: Two-dimensional numerical simulation of natural convection in a square cavity. *Mechanika* **23**(4), 545–551 (2017)
7. Dimitrienko, Y.I., Li, S.: Numerical simulation of MHD natural convection heat transfer in a square cavity filled with Carreau fluids under magnetic fields in different directions. *Comp. Appl. Math.* **39**, 252 (2020)
8. Elguennouni, Y., Hssikou, M., Baliti, J., Alaoui, M.: Cavity inclination effect on convection flow in a triangular geometry. *Heat Transf.* **52**(4), 3253–3279 (2023)
9. Baliti, J., Elguennouni, Y., Hssikou, M., Alaoui, M.: Simulation of natural convection by multirelaxation time lattice Boltzmann method in a triangular enclosure. *Fluids* **7**(2), 74 (2022)
10. Basak, T., Roy, S., Babu, S.K., Balakrishnan, A.R.: Finite element analysis of natural convection flow in an isosceles triangular enclosure due to uniform and non-uniform heating at the side walls. *Int. J. Heat Mass Transf.* **51**, 4496–4505 (2008)
11. Kent, E.F.: Numerical analysis of laminar natural convection in isosceles triangular enclosures. *Proc. Inst. Mech. Eng. Part D J. Mech. Eng. Sci.* **223**, 1157–1169 (2009)
12. Saleem, K.B., Alshara, A.K.: Natural convection in a triangular cavity filled with air under the effect of external air stream cooling. *Heat Transf. Res.* **48**, 3186–3213 (2019)
13. Lyican, L., Bayazitoglu, Y., Witte, L.C.: An analytical study of natural convective heat transfer within a trapezoidal enclosure. *J. Heat Transf.* **102**, 640–647 (1980)
14. Kuyper, R.A., Hoogendoorn, C.J.: Laminar natural convection flow in trapezoidal enclosures. *Numer. Heat Transf. A* **28**, 55–67 (1995)
15. Basak, T., Ramakrishna, D., Roy, S., Matta, A., Pop, I.: A comprehensive heatline based approach for natural convection flows in trapezoidal enclosures: effect of various walls heating. *Int. J. Therm. Sci.* **50**, 1385–1404 (2011)

16. Dutta, S., Goswami, N., Biswas, A.K., Pati, S.: Numerical investigation of magnetohydrodynamic natural convection heat transfer and entropy generation in a rhombic enclosure filled with Cu-water nanofluid. *Int. J. Heat Mass Transf.* **136**, 777–798 (2019)
17. Nia, S.N., Rabiee, F., Rashidi, M.M., Kwang, T.M.: Lattice Boltzmann simulation of natural convection heat transfer of a nanofluid in a L-shape enclosure with a baffle. *Results Phys.* **19**, 103413 (2020)
18. Masoud Seyyedi, S., Dogonchi, A.S., Hashemi-Tilehnoee, M., Waqas, M., Ganji, D.D.: Entropy generation and economic analyses in a nanofluid filled L-shaped enclosure subjected to an oriented magnetic field. *Appl. Therm. Eng.* **168**, 114789 (2020)
19. Raizah, Z.A., Ahmed, S.E., Aly, A.M.: ISPH simulations of natural convection flow in E-enclosure filled with a nanofluid including homogeneous/heterogeneous porous media and solid particles. *Int. J. Heat Mass Transf.* **160**, 120153 (2020)
20. Keramat, F., Dehghan, P., Mofarahi, M., Lee, C.H.: Numerical analysis of natural convection of alumina–water nanofluid in H-shaped enclosure with a V-shaped baffle. *J. Taiwan Inst. Chem. Eng.* **111**, 63–72 (2020)
21. Liu, W., Shahsavari, A., Barzinjy, A.A., Al-Rashed, A.A., Afrand, M.: Natural convection and entropy generation of a nanofluid in two connected inclined triangular enclosures under magnetic field effects. *Int. Commun. Heat Mass Transf.* **108**, 104309 (2019)
22. Roy, N.C., Gorla, R.S.R.: Natural convection of a chemically reacting fluid in a concentric annulus filled with non-Darcy porous medium. *Int. J. Heat Mass Transf.* **127**, 513–525 (2018)
23. Saha, L.K., Bala, S.K., Roy, N.C.: Natural convection flow in a fluid-saturated non-darcy porous medium within a complex wavy wall reactor. *J. Therm. Anal. Calorim.* **146**, 325–340 (2021)
24. Sivasankaran, S., Bhuvaneshwari, M., Alzahrani, A.K.: Numerical simulation on convection of non-Newtonian fluid in a porous enclosure with non-uniform heating and thermal radiation. *Alex. Eng. J.* **59**, 3315–3323 (2020)
25. Alsabery, A.I., Chamkha, A.J., Saleh, H., Hashim, I., Chanane, D.B.: Darcian natural convection in an inclined trapezoidal cavity partly filled with a porous layer and partly with a nanofluid layer. *Sains Malaysiana* **46**(5), 803–815 (2017)
26. Roy, N.C.: Natural convection of nanofluids in a square enclosure with different shapes of inner geometry. *Phys. Fluids* **30**, 113605 (2018)
27. Parvin, S., Roy, N.C., Saha, L.K., Siddiqua, S.: Heat transfer characteristics of nanofluids from a sinusoidal corrugated cylinder placed in a square cavity. *Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci.* **236**(5), 2617–2630 (2022)
28. Saha, L.K., Bala, S.K., Roy, N.C.: Natural convection of dusty nanofluids within a concentric annulus. *Eur. Phys. J. Plus.* **135**, 732 (2020)
29. Chandra Roy, N.: Flow and heat transfer characteristics of a nanofluid between a square enclosure and a wavy wall obstacle. *Phys. Fluids* **31**, 082005 (2019)
30. Venkatadri, K., Bég, O.A., Rajarajeswari, P., Prasad, V.R., Subbarao, A., Khan, B.M.H.: Numerical simulation and energy flux vector visualization of radiative-convection heat transfer in a porous triangular enclosure. *J. Porous Media* **23**(12), 1187–1199 (2020)
31. Brinkman, H.C.: The viscosity of concentrated suspensions and solutions. *J. Chem. Phys.* **20**, 571–581 (1952)
32. Khanafer, K., Vafai, K., Lightstone, M.: Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. *Int. J. Heat Transf.* **46**, 3639–3653 (2003)
33. Oztop, H.F., Abu-Nada, E.: Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. *Int. J. Heat Fluid Flow* **29**, 1326–1336 (2008)
34. Abu-Nada, E., Oztop, H.F.: Effects of inclination angle on natural convection in enclosures filled with Cu-water nanofluid. *Int. J. Heat Fluid Flow* **30**, 669–678 (2009)
35. Muthamilselvan, M., Kandaswamy, P., Lee, L.: Heat transfer enhancement of copper-water nanofluids in lid-driven enclosure. *Commun. Nonlinear Sci. Numer. Simul.* **15**, 1501–1510 (2010)
36. Nield, D.A., Bejan, A.: *Convection in porous media*, 4th edn. Springer, New York (2013)
37. Yu, W., France, D.M., Routbort, J.L., Choi, S.U.S.: Review and comparison of nanofluid thermal conductivity and heat transfer enhancements. *Heat Transf. Eng.* **29**, 432–460 (2008)
38. Sheremet, M.A., Pop, I., Bachok, N.: Effect of thermal dispersion on transient natural convection in a wavy-walled porous cavity filled with a nanofluid: Tiwari and Das’ nanofluid model. *Int. J. Heat Mass Transf.* **92**, 1053–1060 (2016)
39. Murthy, P.V.S.N., Singh, P.: Effect of viscous dissipation on a non-Darcy natural convection regime. *Int. J. Heat Mass Transf.* **40**(6), 1251–1260 (1997)
40. Sun, Q., Pop, I.: Free convection in a triangle cavity filled with a porous medium saturated with nanofluids with flush mounted heater on the wall. *Int. J. Therm. Sci.* **50**, 2141–2153 (2011)

41. Chamkha, A.J., Ismael, M.A.: Conjugate heat transfer in a porous cavity filled with nanofluids and heated by a triangular thick wall. *Int. J. Therm. Sci.* **67**, 135–151 (2013)
42. Vedavathi, N., Venkatadri, K., Fazuruddin, Sd., Raju, G.S.S.: Natural convection flow in semi-trapezoidal porous enclosure filled with alumina-water nanofluid using Tiwari and Das' nanofluid model. *Eng. Trans.* **70**(4), 303–318 (2022)
43. Poulidakos, D., Bejan, A.: Numerical study of transient high rayleigh number convection in an attic-shaped porous layer. *J. Heat Transf.* **105**(3), 476 (1983)

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