

4. INVERTERS

* Inverter converts fixed dc input voltage to variable Ac output voltage.

* The dc input voltage to the inverter can be obtained from either dc supply (or) battery (or) through rectifier.

* The output of the ideal inverter should be sinusoidal but practically it is not sinusoidal and contains harmonics.

* Practically the output of the inverter is either square (or) quasi square wave.

* There are two types of inverters.

(i) Voltage source inverter (VSI)

(or)
Voltage fed inverter.

(ii) Current source inverter

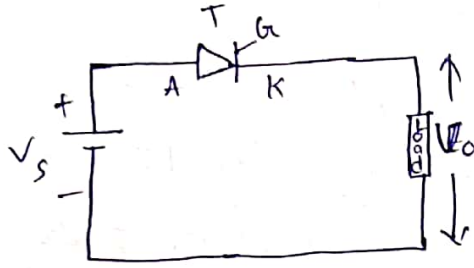
(or)
Current fed inverter.

* when input voltage is maintained constant then the inverter is called voltage source inverter.

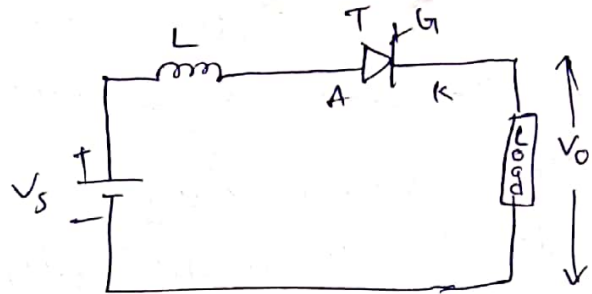
* when input current is maintained constant then the inverter is called current source inverter.

Applications:

- 1) Variable speed AC motor drive
- 2) Induction heating
- 3) Uninterruptible Power Supplies.
- 4) Stand by aircraft Power Supplies.
- 5) HVDC Transmission lines etc.



VSI



CSI

Inverters are also classified as follows depending upon the connection of commutating components with the main circuit.

- 1) Series Inverters.
- 2) Parallel Inverters.
- 3) Bridge Inverters.

1) Series Inverter:

In the series inverters, the commutating elements L and C are connected in series with the load. This constitutes a series $R-L-C$ resonance circuit. This type of thyristorised inverter produces an approximate sinusoidal waveform at high output frequency whose range is given as $200\text{Hz} - 100\text{kHz}$.

These are commonly used in relatively fixed output applications such as ultrasonic generators, induction heating, sonar transmitter, fluorescent tubes etc.

T_1 & T_2 carry load current in +ve & -ve half cycles.

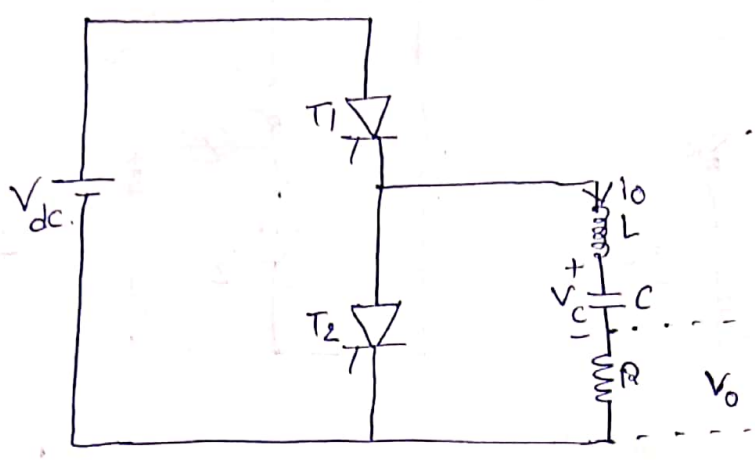


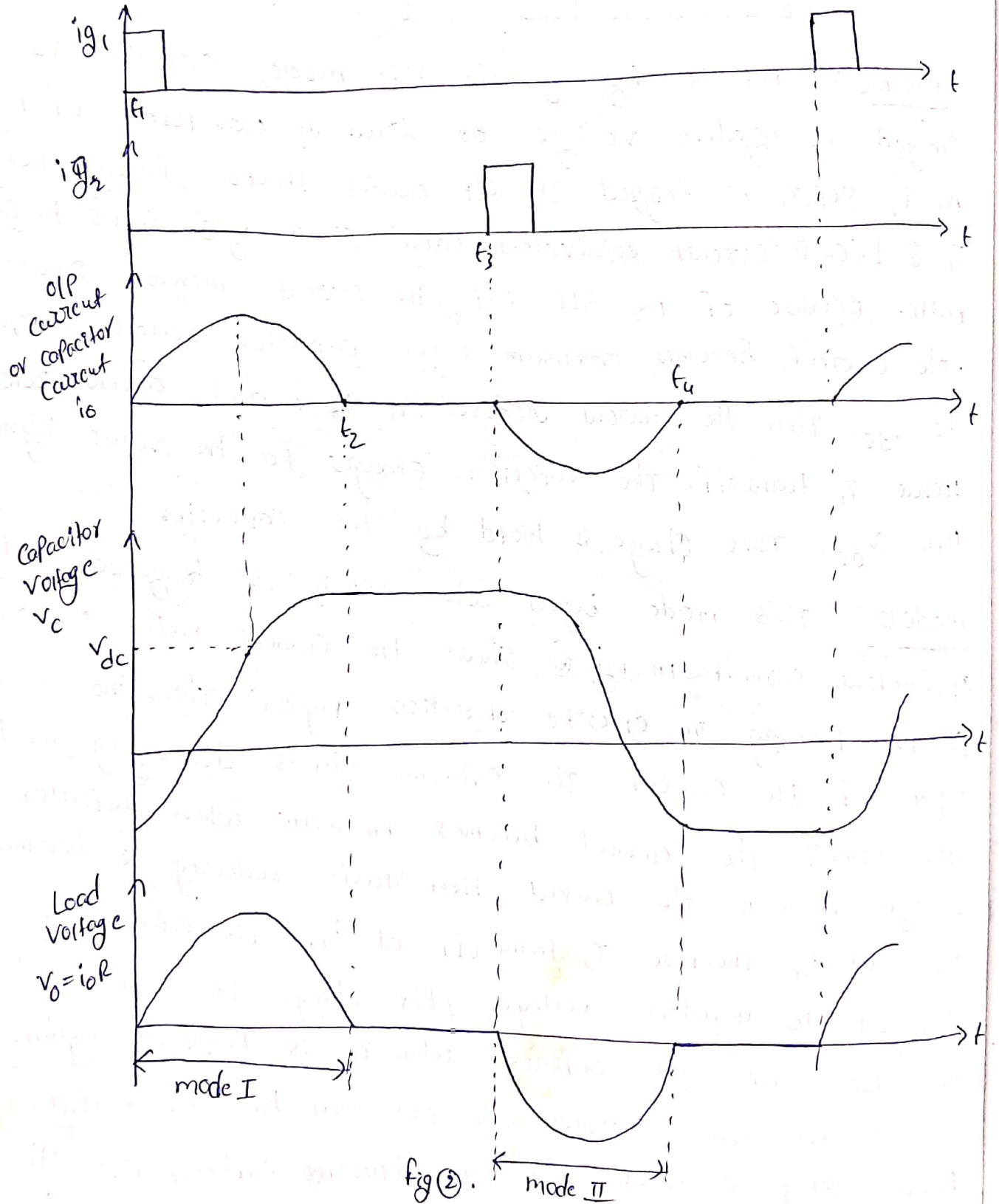
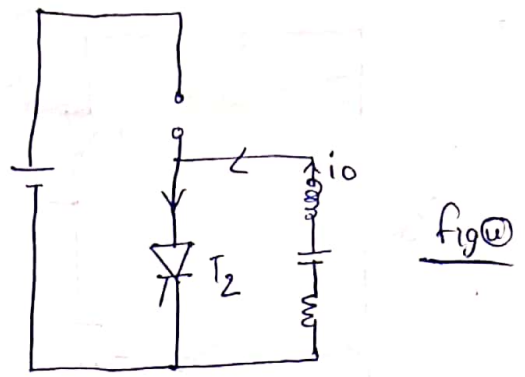
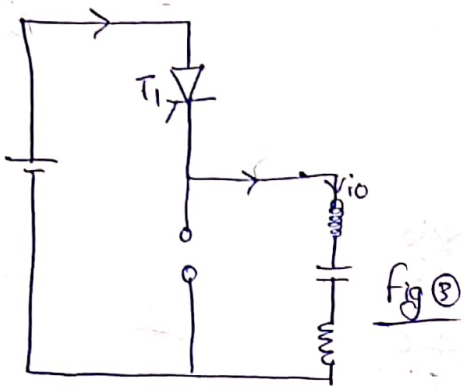
Fig 1:

mode 1: At the beginning of this mode, capacitor is charged to negative voltage as shown in waveforms of fig 2. At t_1 , SCR T_1 is triggered. The old current starts flowing through T_1 & L-C-R circuit equivalent circuit - I in fig 3 shows the current path. Because of the RLC CK, the current increases sinusoidally. The current becomes maximum when capacitor voltage is equal to V_{dc} . Then the current reduces. At t_2 current become zero. Hence T_1 turns off. The capacitor charges to the value higher than V_{dc} . This charge is hold by the capacitor.

mode 2: This mode begins when SCR T_2 is triggered at t_3 . equivalent circuit in fig 4 shows the current path. The current starts flowing in opposite direction. fig 5 shows the -ve half cycle of the current. The capacitor starts discharging in the RLC circuit. The current becomes maximum when capacitor voltage is zero. The current then starts reducing & becomes zero at t_4 . Therefore T_2 turns off at t_4 . The capacitor is charged to negative voltage. This charge is hold by the capacitor. The cycle repeats when T_1 is triggered again.

The series resonant RLC CK must be underdamped.

Hence R & C must satisfy following condition $R < \sqrt{\frac{4L}{C}}$

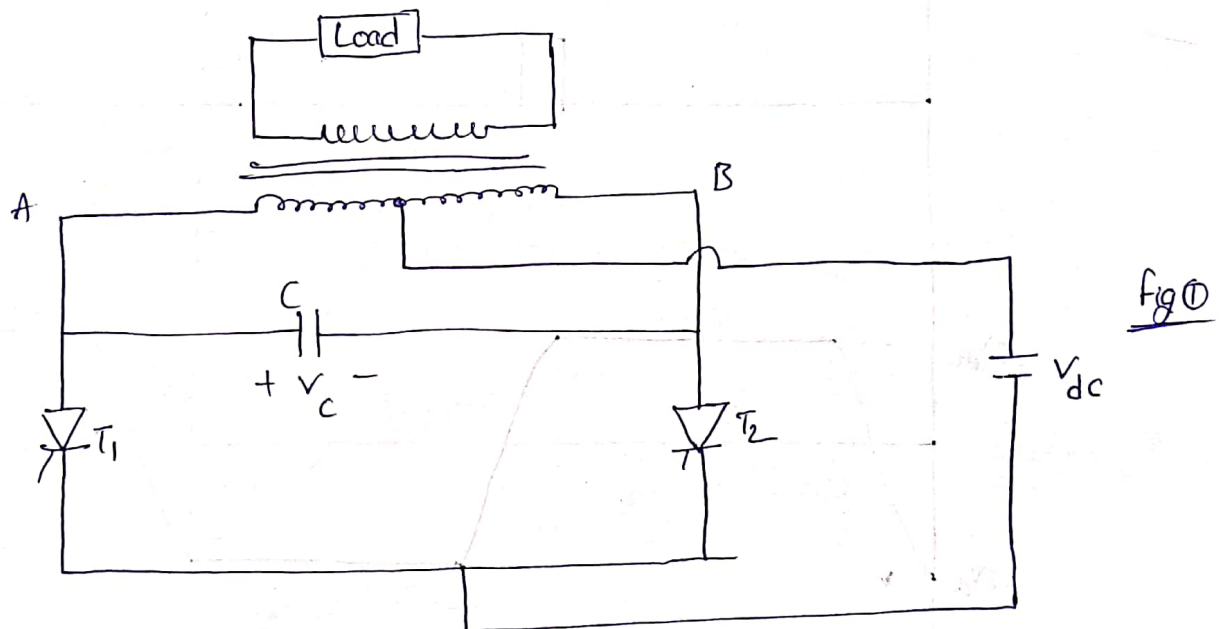


Q) Parallel inverter:

In Parallel inverter the commutating elements are connected across the load.

It consists of two thyristors T_1 & T_2

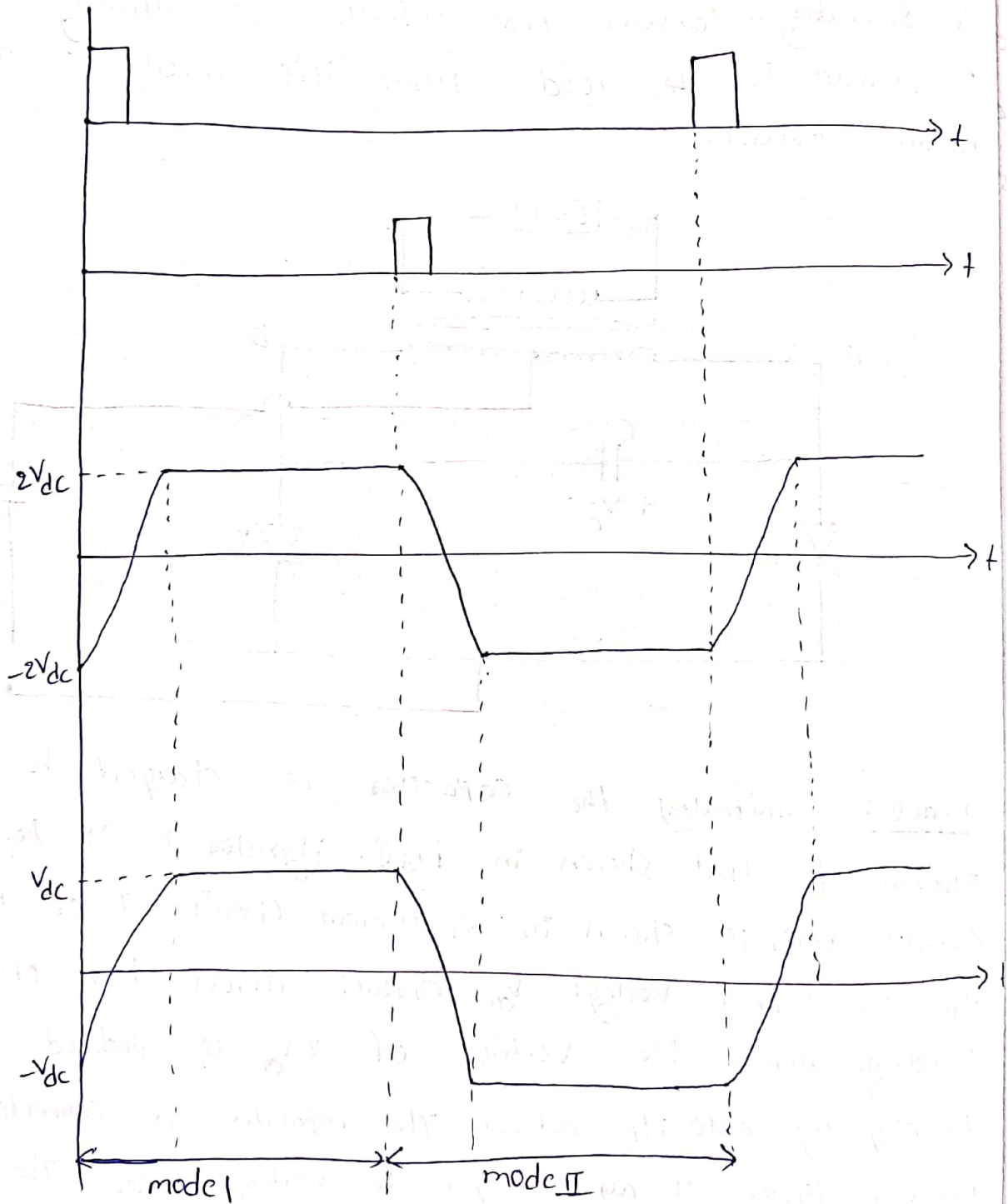
Capacitor 'C' is the commutating capacitor. Capacitor is connected across the TLF primary. Load is connected across the secondary. Observe that capacitor is indirectly connected in parallel to the load. Hence this inverter is called Parallel inverter.



mode 1: Initially the capacitor is charged to the polarity opposite to that shown in Fig 1. Thyristor T_1 is triggered. The current path is shown in equivalent circuit - I of Fig 2. Observe that the supply voltage V_{dc} appears across half of the primary winding. Hence the voltage of $2V_{dc}$ is induced across the primary by auto TLF action. The capacitor is connected across primary. Hence it also charges to voltage $2V_{dc}$. The capacitor remains charged to this level.

mode ②: To initiate negative half cycle, thyristor T_2 is triggered.

As soon as T_2 is triggered, a capacitor voltage of $-2V_{dc}$ is applied across T_1 . Hence T_1 immediately turns off. The load current starts flowing through other half of the primary winding. This is shown in equivalent circuit II of fig ①. The load voltage becomes negative as shown in the waveforms.



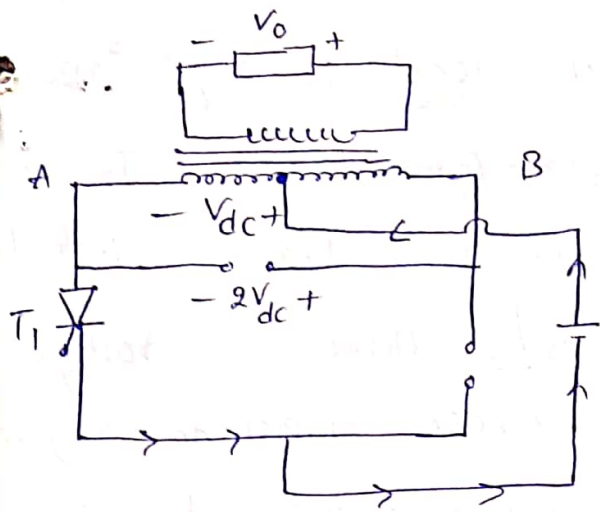


Fig 3

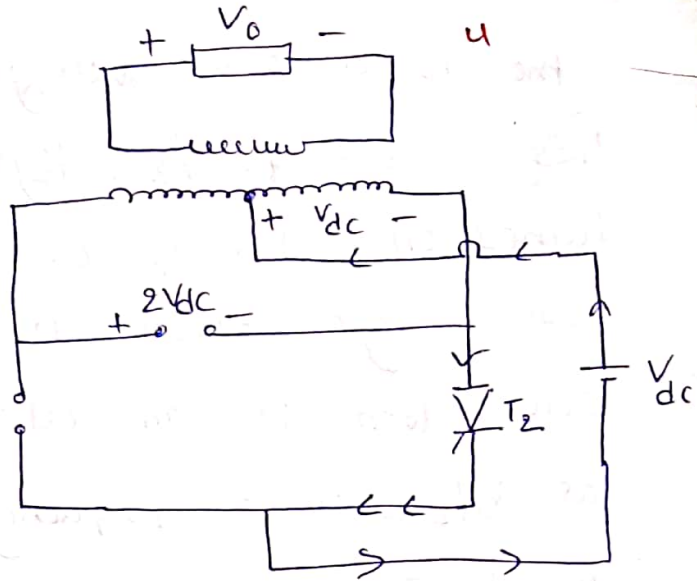


Fig 4

Advantages:

- 1) o/p voltage is square wave, which is better than series inverter
- 2) simple commutating circuit
- 3) switching frequency is higher.

Disadvantages:

- 1) Heavy T/F is required to carry load current
- 2) Large amount of energy is trapped in commutating capacitor. It is to be removed with the help of additional feedback diodes.

* Based on connection of commutating elements inverters are classified into three.

1. 1- ϕ bridge inverter
 2. 1- ϕ Series inverter
 3. 1- ϕ parallel inverter
- half bridge inverter:

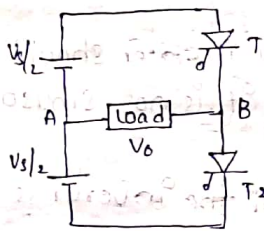
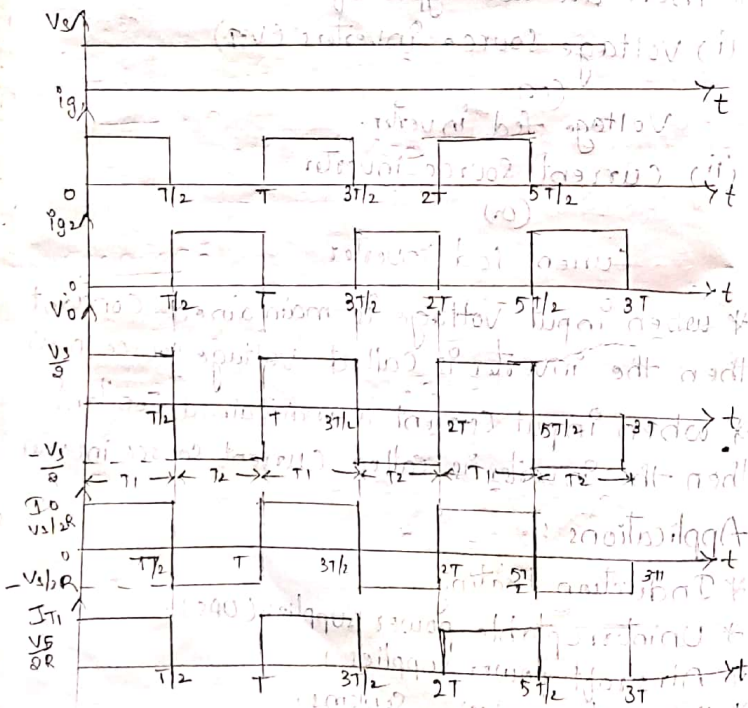


Fig 1: 1- ϕ half bridge inverter



$0 - \frac{T}{2}$: $T_1 - ON$; $\frac{T}{2} - T$: $T_2 - ON$

$$V_o = \frac{V_s}{2}$$

$$I_o = \frac{V_o}{R} = \frac{V_s}{2R}$$

$$i_o = B - A$$

$$V_{rms} = \sqrt{\frac{1}{T} \left[\int_0^{T/2} V_o^2 dt + \int_{T/2}^T (-V_o)^2 dt \right]}$$

$$V_{rms} = \frac{1}{T} \left[\int_0^{T/2} \left(\frac{V_s}{2}\right)^2 dt + \int_{T/2}^T \left(-\frac{V_s}{2}\right)^2 dt \right]$$

$$= \frac{1}{T} \left(\frac{V_s}{2}\right)^2 \left[\left(\frac{T}{2} - 0\right) + \left(T - \frac{T}{2}\right) \right]$$

$$V_{rms} = \frac{1}{T} \left(\frac{V_s}{2}\right)^2 T$$

$$V_{rms} = \frac{V_s}{2}$$

$$I_{rms} = \frac{V_{rms}}{R}$$

3. By Fourier series the output voltage can be expressed in sinusoidal as

$$V_o = \sum_{n=1,3,5} \frac{2V_s}{n\pi} \sin n\omega t$$

$V_o = 0$ for $n=2, 4, 6, \dots$

4. For n^{th} harmonic output voltage

$$V_o = \frac{2V_s}{n\pi} \sin n\omega t$$

5. Peak value of output voltage for n^{th} harmonic

$$= \frac{2V_s}{n\pi}$$

6. Peak value of fundamental = $\frac{2V_s}{\pi}$

7. RMS Value of fundamental = $\frac{2V_s}{\pi\sqrt{2}}$

8. Rms Value of output voltage for n^{th} harmonic

$$= \frac{2V_s}{n\pi\sqrt{2}} = \frac{0.9V_s}{n}$$

9. output power $P_o = V_{orms} \cdot I_{orms} = \frac{V_{orms}^2}{R}$

10. Peak thyristor current $I_{TP} = \frac{V_s}{2R}$

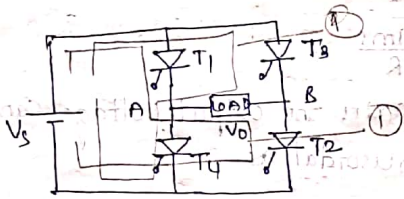
11. Average thyristor current $I_{TA} = \frac{1}{T} \int_0^T I_T dt$

$$\Rightarrow I_{TA} = \frac{1}{T} \int_0^{\pi/2} \left(\frac{V_s}{2R} \right) dt$$

$$= \frac{1}{T} \left(\frac{V_s}{2R} \right) \frac{T}{2}$$

$$I_{TA} = \frac{1}{2} \times I_{TP}$$

1- ϕ full bridge inverter:



Operation:

$$0 - \frac{T}{2}$$

$$\frac{T}{2} - T$$

$$T_1 - ON$$

$$T_2 - ON$$

$$V_o = V_s$$

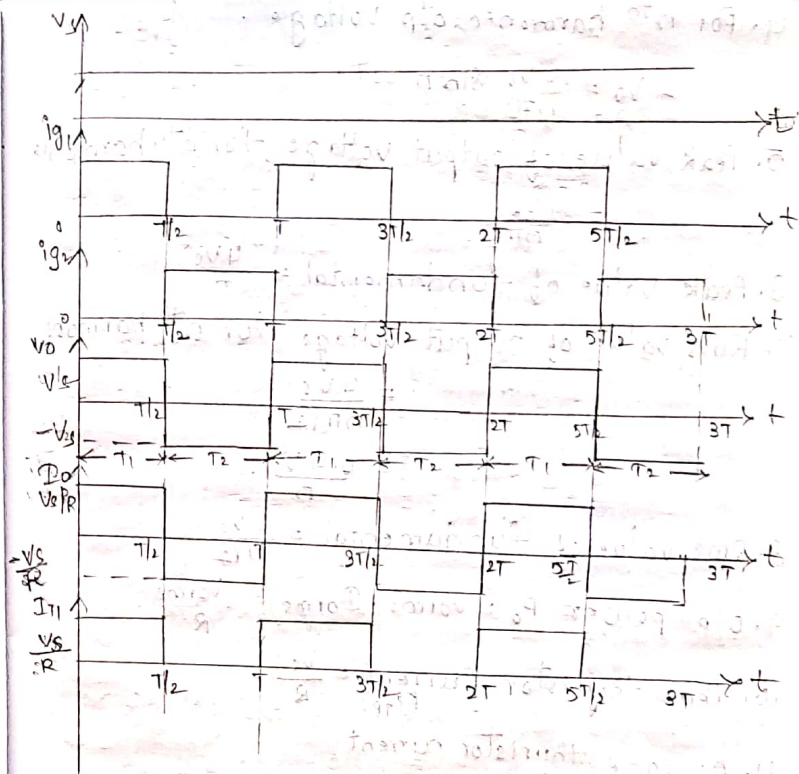
$$V_o = -V_s$$

$$I_o = \frac{V_o}{R} = \frac{V_s}{R}$$

$$I_o = \frac{V_o}{R} = -\frac{V_s}{R}$$

$$i_o = A - B$$

$$i_o = B - A$$



$$1. V_{orms} = \sqrt{\frac{1}{T} \int_0^T V_o^2 dt}$$

$$V_{orms}^2 = \frac{1}{T} \left[\int_0^{\pi/2} V_s^2 dt + \int_{\pi/2}^{\pi} (-V_s)^2 dt \right]$$

$$= \frac{V_s^2}{T} \left[\frac{T}{2} + \frac{T}{2} \right]$$

$$V_{orms}^2 = \frac{V_s^2}{2} \times T$$

$$V_{orms} = \frac{V_s}{\sqrt{2}}$$

$$2. I_{orms} = \frac{V_{orms}}{R}$$

3. By Fourier Series output voltage can be expressed as $V_o = \sum_{n=1,3,5}^{\infty} \frac{4V_s}{n\pi} \sin n\omega t$

4. For n^{th} harmonic o/p voltage

$$V_o = \frac{4V_s}{n\pi} \sin n \omega t$$

5. Peak value of output voltage for n^{th} harmonic

$$= \frac{4V_s}{n\pi}$$

6. Peak value of fundamental = $\frac{4V_s}{\pi}$

7. RMS value of output voltage for n^{th} harmonic

$$= \frac{4V_s}{n\pi\sqrt{2}}$$

$$= 0.9 \frac{V_s}{n}$$

8. RMS value of fundamental = $\frac{4V_s}{\pi\sqrt{2}}$

9. o/p power $P_o = V_{\text{orms}} \cdot I_{\text{orms}} = \frac{V_{\text{orms}}^2}{R}$

10. Peak thyristor current = $\frac{V_s}{R}$

11. Average thyristor current

$$I_{TA} = \frac{1}{T} \int_0^T I_T dt = \frac{1}{T} \left[\int_0^{\pi/2} \frac{V_s}{R} dt + \int_{\pi/2}^T 0 dt \right]$$

$$I_{TA} = \frac{1}{2} I_{TP}$$

* 1- ϕ half bridge inverter has a dc input of 48V, $R = 4.8 \Omega$. Determine (i) RMS output voltage (ii) RMS value of fundamental component (iii) output power (iv) peak and average thyristor currents (v) total harmonic distortion.

Sol: Given that,

$$V_s = 48 \text{ V}$$

$$R = 4.8 \Omega$$

$$(i) V_{\text{orms}} = \frac{V_s}{2} = \frac{48}{2} = 24 \text{ V}$$

$$(ii) V_{1\text{rms}} = \frac{2V_s}{\pi\sqrt{2}} = \frac{2 \times 48}{\pi\sqrt{2}} = 21.6 \text{ V}$$

$$(iii) P_o = \frac{V_{\text{orms}}^2}{R} = \frac{(24)^2}{4.8} = 120 \text{ W}$$

$$(iv) I_{TP} = \frac{V_s}{2R} = \frac{48}{2(4.8)} = 5 \text{ A}$$

$$I_{TA} = \frac{1}{2} I_{TP} = \frac{1}{2} \times 5 = 2.5 \text{ A}$$

(v) Total harmonic distortion

$$THD = \frac{\sqrt{V_{\text{orms}}^2 - V_{1\text{rms}}^2}}{V_{1\text{rms}}} = \sqrt{\frac{V_{\text{orms}}^2 - V_{1\text{rms}}^2}{V_{1\text{rms}}^2}}$$

$$\Rightarrow THD = \sqrt{\left(\frac{V_{\text{orms}}}{V_{1\text{rms}}}\right)^2 - 1}$$

$$= \sqrt{\left(\frac{24}{21.6}\right)^2 - 1}$$

$$= 0.484$$

$$THD = 48.4\%$$

* 1- ϕ full bridge inverter has dc input of 24 volts, $R = 4.8 \Omega$. Determine (i) RMS output voltage (ii) RMS value of fundamental component (iii) output power (iv) peak and average thyristor currents (v) total harmonic distortion.

Sol: Given that,

$$V_s = 24 \text{ V}$$

$$R = 4.8 \Omega$$

$$(i) V_{\text{orms}} = \frac{V_s}{2} = 12 \text{ V}$$

$$(ii) V_{1\text{rms}} = \frac{4V_s}{\pi\sqrt{2}} = \frac{4 \times 24}{\pi\sqrt{2}} = 17.60 \text{ V}$$

$$(iii) P_o = \frac{V_{\text{orms}}^2}{R} = \frac{(12)^2}{4.8} = 30 \text{ W}$$

$$(iv) I_{TP} = \frac{V_s}{R} = 5 A$$

$$I_{TA} = \frac{1}{2} I_{TP} = \frac{1}{2} \times 5 = 2.5 A$$

$$(v) THD = \frac{\sqrt{V_{orms}^2 - V_{irms}^2}}{V_{irms}} = \sqrt{\frac{V_{orms}^2 - V_{irms}^2}{V_{irms}^2}}$$

$$\Rightarrow THD = \sqrt{\left(\frac{V_{orms}}{V_{irms}}\right)^2 - 1}$$

$$= \sqrt{\left(\frac{1.414}{1}\right)^2 - 1}$$

$$THD = 0.503$$

Voltage Control techniques in 1- ϕ inverter:

Actually AC loads may require constant (or) adjustable voltage at its input terminals. When such AC loads are fed through inverters, the output voltage of the inverter needs to be controlled as per the load requirement.

There are three methods to control the output voltage of the inverter.

1. External control of AC output voltage.
2. External control of dc input voltage.
3. Internal control of inverter.

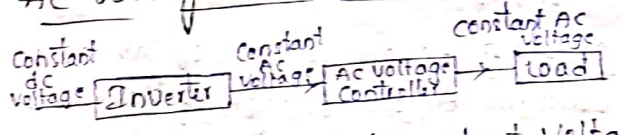
External control of AC output voltage:

In this method the output voltage of the inverter can be controlled at its output terminals.

There are two methods of external control of AC output voltage.

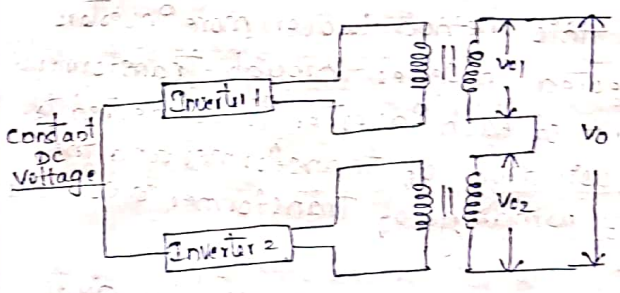
- * AC Voltage control method.
- * Series inverter control method.

AC voltage control method:



In this method the output voltage of the inverter is controlled at inverter output terminals through AC voltage controllers.

Series inverter control method:



Case (i):

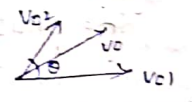
when $\theta = 0$, $\cos \theta = 1$

$$V_0 = \sqrt{V_{01}^2 + V_{02}^2 + 2V_{01}V_{02}}$$

$$V_0 = (V_{01} + V_{02})^2$$

$$V_0 = V_{01} + V_{02}$$

If $V_{01} = V_{02}$, $V_0 = 2V_{01} \cos \theta + V_{02}$



$$V_0 = \sqrt{V_{01}^2 + V_{02}^2 + 2V_{01}V_{02} \cos \theta}$$

Case (ii)

when $\theta = \pi$, $\cos \theta = -1$

$$V_0 = \sqrt{V_{01}^2 + V_{02}^2 - 2V_{01}V_{02}}$$

$$V_o = \sqrt{(V_{o1} - V_{o2})^2}$$

$$V_o = V_{o1} - V_{o2}$$

If $V_{o1} = V_{o2}$, $V_o = 0$

V_{o1} = Constant AC o/p voltage of inverter-1
 V_{o2} = Constant AC o/p voltage of inverter-2

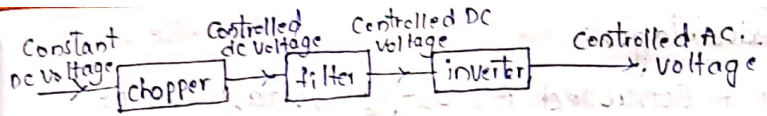
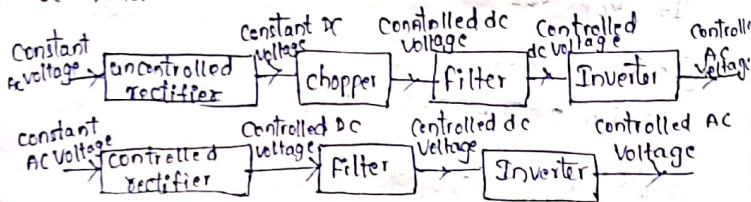
where θ = angle between V_{o1} and V_{o2} ; by varying angle θ the output voltage of the inverter V_o can be controlled but θ can be varied by varying the firing angles of inverter.

In this method two (or) more inverters are connected in series through transformers. The output of each inverter is connected to primary winding of transformer and the secondary windings of transformer are connected in series.

The phasor sum of V_{o1} and V_{o2} gives resultant voltage V_o .

2. External control of dc input voltage:

In this method the output voltage of the inverter can be controlled at its input terminals.



Disadvantages:

- * In order to reduce the ripples, filter circuits are used. This will increase the cost, size and weight of the filter.
- * In order to obtain controlled AC^{o/p} voltage of inverter, it requires more number of components.
- * As number of components are more, it is very difficult to control the output voltage of the inverter because more number of components leads to more power loss and thus decreases the efficiency.

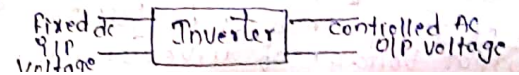
3. Internal control of inverter:

In this method the output voltage of the inverter can be controlled within the inverter itself.

The most efficient method of controlling the output voltage of inverter is pulse width modulation control (PWM).

Pulse width modulation control:

In this a fixed DC input voltage is given as input to the inverter. A controlled AC o/p voltage is obtained directly by adjusting on and off periods of inverter power semiconducting devices.



Advantages:

- * A controlled AC output voltage can be obtained directly without using any additional components.
- * Higher order harmonics can be eliminated along with the control of output voltage. Lower order harmonics can be minimized by using filter circuits.

Disadvantages:

- * Thyristors used must have low turn on and turn off times i.e. inverter grade SCR's should be used to control the output voltage of inverter.
- * It is very expensive.

Pulse width modulation techniques (or) harmonic reduction techniques:

- * Single pulse width modulation technique (SPWM).
- * Multiple pulse width modulation (MPWM).
- * Sinusoidal pulse width modulation (Sin PWM).

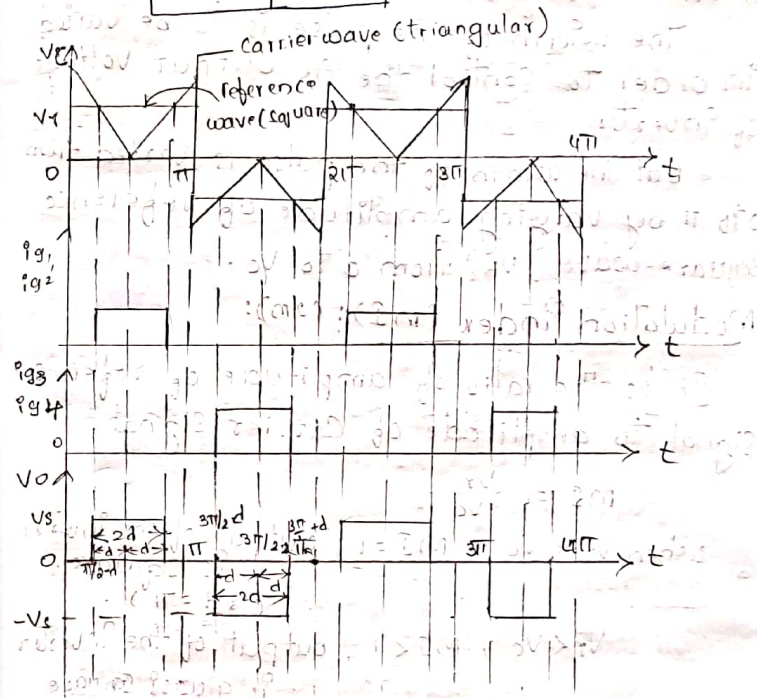
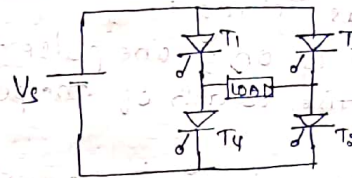
SPWM:

If inverter uses pulse width modulation controls then the inverter are called PWM inverter.

Pulse width modulation techniques are characterised by constant amplitude pulse but width of the pulse is to be varied to control the AC output voltage of inverter and the reduction

the harmonic content in the output.

SPWM:



Triggering pulses (I_{g1} , I_{g2} , I_{g3} and I_{g4}) are used generated at the intersection point of reference signal and carrier signal i.e. triggering pulses to the SCR are generated by comparing reference signal and carrier signal. The comparison is done by using comparator. The generated triggering pulses will turn on the SCR and the output voltage will available across

the load:

SPWM uses reference wave as square and carrier wave as triangle.

In SPWM there is only one pulse for half cycle. Assume the width of the pulse is αd .

The width of the pulse is to be varied in order to control the AC output voltage of inverter.

But the width of the pulse is varied from 0 to π by varying amplitude of reference square wave (V_r) from 0 to V_c .

Modulation Index (MI): (MI):

It is the ratio of amplitude of reference signal to amplitude of carrier signal.

$$MI = \frac{V_r}{V_c}$$

when $V_r = V_c$, $MI = 1$, output of the inverter is square. ($\alpha d = \pi$).

$V_r < V_c$, $MI < 1$, output of the inverter is quasi square ($\alpha d \neq \pi$).

1. By Fourier Series the output voltage of the inverter expressed in terms of sine wave as

$$V_o = \sum_{n=1,3,5} \frac{4V_s}{n\pi} \sin \frac{n\pi}{2} \sin n\alpha d \sin n\omega t$$

$$V_o = 0, \text{ for } n = 2, 4, 6, \dots$$

} quasi square ($\alpha d \neq \pi$)

For square $\alpha d = \pi$, $d = \frac{\pi}{2}$

$$V_o = \sum_{n=1,3,5} \frac{4V_s}{n\pi} \sin n\omega t \quad \left. \vphantom{\sum} \right\} \text{Square wave } (\alpha d = \pi)$$

$$V_o = 0, \text{ for } n = 2, 4, 6, \dots$$

2. The output voltage of the inverter for n^{th} harmonic

$$V_o = \frac{4V_s}{n\pi} \sin \frac{n\pi}{2} \sin n\alpha d \sin n\omega t \quad \left. \vphantom{\sum} \right\} \text{quasi square } (\alpha d \neq \pi)$$

$$V_o = 0 \text{ for } n = 2, 4, 6, \dots$$

for square

$$V_o = \frac{4V_s}{n\pi} \sin n\omega t \text{ for } n = 1, 3, 5 \quad \left. \vphantom{\sum} \right\} \text{square } (\alpha d = \pi)$$

$$V_o = 0, \text{ for } n = 2, 4, 6, \dots$$

3. Peak value of output for n^{th} harmonic.

$$V_o = \frac{4V_s}{n\pi} \sin \frac{n\pi}{2} \sin n\alpha d \quad \left. \vphantom{\sum} \right\} \text{quasi square } (\alpha d \neq \pi)$$

$$V_o = \frac{4V_s}{n\pi} \quad \left. \vphantom{\sum} \right\} \text{Square } (\alpha d = \pi)$$

4. Peak value of output for fundamental.

$$V_o = \frac{4V_o}{\pi} \sin d \quad \text{quasi square}$$

$$V_o = \frac{4V_s}{\pi} \quad \text{square}$$

5. To eliminate n^{th} harmonic the value of αd should be equal to π .

$$\text{i.e. } n\alpha d = \pi$$

$$d = \frac{\pi}{n}$$

$$\alpha d = \frac{2\pi}{n}$$

width of the pulse to eliminate n^{th} harmonic.

To eliminate third harmonic, the width of the pulse = $\frac{2\pi}{3} = 120^\circ$.

To eliminate fifth harmonic, the width of the pulse = $\frac{2\pi}{5} = 72^\circ$.

$$6. V_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} V_o^2 dt}$$

$$= \frac{1}{2\pi} \left[\int_{\frac{\pi}{5}-d}^{\frac{\pi}{5}+d} V_s^2 dt + \int_{\frac{3\pi}{5}-d}^{\frac{3\pi}{5}+d} (-V_s)^2 dt \right]$$

$$= \frac{1}{2\pi} \left[\int_{\frac{\pi}{5}-d}^{\frac{\pi}{5}+d} V_s^2 dt + \int_{\frac{3\pi}{5}-d}^{\frac{3\pi}{5}+d} V_s^2 dt \right]$$

$$= \frac{2}{2\pi} \left[\int_{\frac{\pi}{5}-d}^{\frac{\pi}{5}+d} V_s^2 dt \right]$$

$$= \frac{V_s^2}{\pi} \left[\frac{\pi}{5} + d - \frac{\pi}{5} + d \right]$$

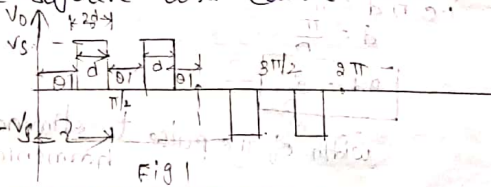
$$V_{rms}^2 = V_s^2 \left[\frac{2d}{\pi} \right]$$

$$V_{rms} = V_s \left[\frac{2d}{\pi} \right]^{1/2}$$

2. MPWM:

In multiple pulse width modulation multiple equidistance pulses for half cycles are present. So this is also called as uniform pulse width modulation.

In multiple pulse width modulation also reference wave is square and carrier wave is triangle.



For simplicity consider 2 pulses for half cycle shown in fig 1.

For 2 pulses for half cycle, there are three equidistance spaces. Similarly for n pulses for half cycles there are (n+1) equidistance spaces. The total width of (n+1) equidistance spaces is given by, spaces width of equidistance space is assumed as θ . The total width of (n+1) equidistance space is given by

$$(N+1)\theta_1 = \pi - \text{Total width of pulse}$$

$$= \pi - 2d$$

$$\theta_1 = \frac{\pi - 2d}{(N+1)}$$

From fig 1 $\theta_2 = \frac{d}{N} = \text{half of pulse width}$.

$$\text{Now } \theta = \theta_1 + \theta_2 = \frac{\pi - 2d}{(N+1)} + \frac{d}{N}$$

$$\theta = \frac{\pi - 2d}{N+1} + \frac{d}{N}$$

The above equation is valid only in case pulses of equal width and are symmetrically spacing.

1. By Fourier Series the output voltage can be expressed in terms of sine wave as

$$V_o = \sum_{n=1,3,5} \frac{8V_s}{n\pi} \sin n\theta \sin \frac{n\pi d}{2} \sin n\omega t$$

$$V_o = 0, \text{ for } n = 2, 4, 6, \dots$$

2. output voltage for nth harmonic,

$$V_o = \frac{8V_s}{n\pi} \sin n\alpha \sin \frac{n\delta}{2} \sin n\omega t, \quad n=1, 3, \dots$$

3. Peak value of output voltage for n^{th} harmonic

$$V_{o\text{np}} = \frac{8V_s}{n\pi} \sin n\alpha \sin \frac{n\delta}{2}$$

4. Peak value of output voltage for fundamental ($n=1$)

$$V_{o\text{op}} = \frac{8V_s}{\pi} \sin \alpha \sin \frac{\delta}{2}$$

5. $n\delta = 2\pi$
 $\delta = \frac{2\pi}{n}$

To eliminate n^{th} harmonic the value of $n\delta$ should be made equal to 2π as given above

6. RMS output voltage $V_{\text{orms}} = \sqrt{\frac{1}{\pi} \int_0^\pi V_o^2 dt}$

$$V_{\text{orms}}^2 = \frac{1}{\pi} \left[\int_{-\frac{\delta}{2}}^{+\frac{\delta}{2}} V_s^2 dt \times 2 \right]$$

$$= \frac{2V_s^2}{\pi} \left[\delta + \frac{\delta}{2} - \delta + \frac{\delta}{2} \right]$$

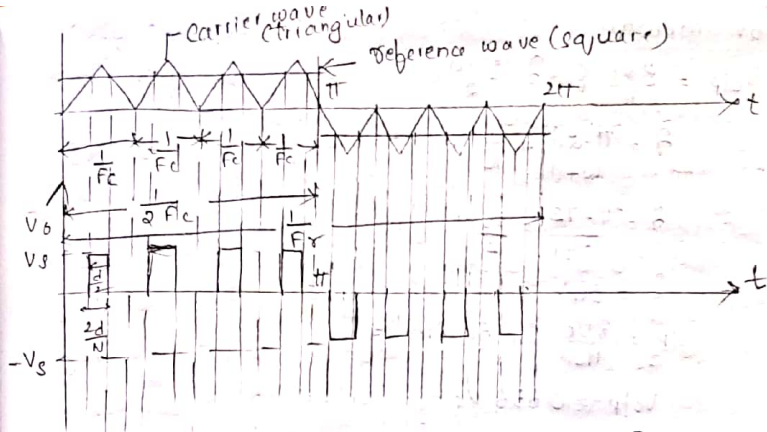
$$V_{\text{orms}}^2 = \frac{2V_s^2}{\pi} \delta$$

$$V_{\text{orms}} = V_s \left(\frac{2\delta}{\pi} \right)^{1/2}$$

from figure given below

$$\frac{1}{2F_r} = 4 \times \frac{1}{F_c}$$

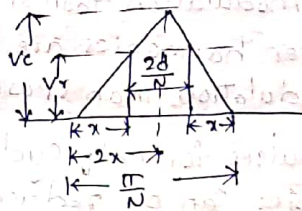
$$4 = \frac{F_c}{F_r}$$



No. of pulses per half cycle (N) = $\frac{F_c}{2F_r}$

when F_r = frequency of reference wave.

F_c = frequency of carrier wave.



width of the pulse ($\frac{2d}{N}$) = $\frac{\pi}{N} - 2x$

$$\frac{V_c}{\frac{\pi}{2N}} = \frac{V_s}{2N}$$

$$x = \frac{V_s}{V_c} \frac{\pi}{2N}$$

width of the pulse ($\frac{2d}{N}$) = $\frac{\pi}{N} - 2 \left(\frac{V_s}{V_c} \frac{\pi}{2N} \right)$

width of the pulse (δ) = $\frac{\pi}{N} \left(1 - \frac{V_s}{V_c} \right)$

for ex: $2d = 72$ $d = 36$

for SPWM (IP/nc)

$$V_{o\text{lp}} = \frac{4V_s}{\pi} \sin \alpha$$

$$V_{o\text{lp}} = \frac{4V_s}{\pi} \sin 36^\circ = 0.748 V_s$$

For MPWM

$$V_{olp} = \frac{8V_s}{\pi} \sin \alpha \sin \frac{\alpha}{2}$$

$$\alpha = \frac{\pi - 2d}{N+1} + \frac{d}{N}$$

$$\alpha = \frac{\pi - 72}{2+1} = \frac{36}{2}$$

$$\alpha = 54^\circ$$

$$V_{olp} = \frac{8V_s}{\pi} \sin 54 \sin \frac{36}{2}$$

$$V_{olp} = 0.636 V_s$$

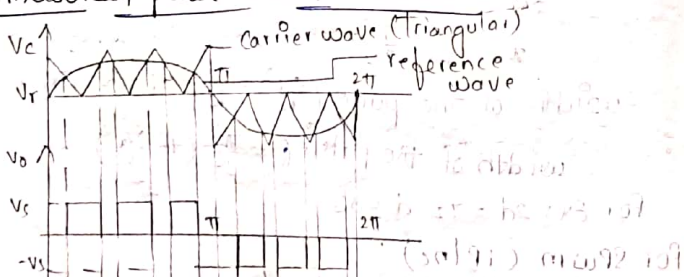
Note:

In MPWM, lower order harmonics can be reduced by using the proper choice of α & N .

Fundamental component of output is low in two pulse width modulation than SPWM. This indicates lower order harmonics are low in two pulse width modulation than SPWM.

As number of pulses for half cycle increases lower order harmonics can be reduced. The higher order harmonics are easily minimized by using filters at the output terminals of inverter.

Sinusoidal pulse width modulation:

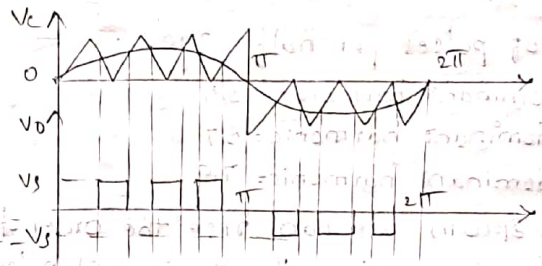


Case ①: Peak of carrier coincidence with zero of reference wave.

$$\frac{1}{2f_r} = 3 \times \frac{1}{f_c}$$

$$3 = \frac{f_c}{2f_r}$$

$$\text{No. of pulses / half cycle } N = \frac{f_c}{2f_r}$$



Case ②: Zero of the carrier coincidence with zero of reference wave.

$$\frac{1}{2f_r} = 4 \frac{1}{f_c}$$

$$4 = \frac{f_c}{2f_r}$$

$$3+1 = \frac{f_c}{2f_r}$$

$$N+1 = \frac{f_c}{2f_r}$$

$$N = \frac{f_c}{2f_r} - 1$$

$$\text{No. of pulses / half cycle } (N) = \frac{f_c}{2f_r} - 1$$

In sinusoidal pulse width modulation, reference wave is sine and carrier wave is triangular. Like MPWM in sinusoidal pulse width modulation, multiple number of pulses per half cycles are present.

But in MPWM the width of each pulse is same where as in sinusoidal pulse width modulation the width of the pulse is sine function of angular position of pulses in each half cycles i.e. sinusoidal.

Pulse width modulation, width of the pulse is not same. In sinusoidal PWM the dominant or significant harmonics are given by:

$$2N \pm 1$$

where

N = no. of pulses per half cycle

$N=2$, dominant harmonic = 3, 5

$N=3$, dominant harmonic = 5, 7

$N=4$, dominant harmonic = 7, 9

Thus in spwm we can rise the order of harmonics by increasing the number of pulses for half cycles. If harmonics belongs to higher order, we can easily eliminate them by using filters.

The rms output voltage of inverter is given

by
$$V_{orms} = V_s \left[\sum_{m=1}^N \frac{P_m}{\pi} \right]^{1/2}$$

P_m = Pulse width of pulse.

* In ^{sinusoidal PWM} inverter, amplitude and frequency for triangular carrier and sinusoidal reference signals are 5V, 1kHz and 1V, 50Hz.

* If zero of the triangle carrier and reference sinusoidal coincides

* What is the modulation index and order of the significant harmonics.

$V_c = 5V$, $f_c = 1kHz$ for Δ 's carrier signal

$V_r = 1V$, $f_r = 50Hz$ for sinusoidal reference signal

Sol: 1. $m_f = \frac{V_r}{V_c} = \frac{1}{5} = 0.2$

w.k.T

$$N = \frac{f_c}{2f_r} - 1 = \frac{1 \times 10^3}{2 \times 50} - 1 = 9$$

2. Order of significant harmonics = $2N \pm 1$
 $= 2(9) \pm 1$
 $= 17, 19$

* If peak of the triangular carrier coincides with zero of the reference of the sinusoidal, what is the modulation index and order of the significant harmonics.

Sol: $m_f = \frac{V_r}{V_c} = \frac{1}{5} = 0.2$

$$N = \frac{f_c}{2f_r} = \frac{1 \times 10^3}{2 \times 50} = 10$$

$2N + 1$

$$2(10) + 1 = 21$$

* In multiple pulse width modulation inverter, the amplitude and frequency for triangular carrier and square reference wave signals are 4V, 6kHz and 1V, 1kHz. Find the no. of pulses for half cycles and pulse width and rms value of output voltage for fundamental.

Sol: $V_c = 4V$, $f_c = 6kHz$ for triangular carrier wave
 $V_r = 1V$, $f_r = 1kHz$ for square reference wave

$$1. N = \frac{f_c}{2f_r} = \frac{6 \times 10^3}{2 \times 1 \times 10^3} = 3$$

2. Pulse width $\left(\frac{2d}{\pi}\right) = \frac{\pi}{\pi} \left(1 - \frac{V_r}{V_c}\right) = \frac{\pi}{3} \left(1 - \frac{1}{4}\right)$
 $\left(\frac{2d}{\pi}\right) = 45$

3. wkt $V_{olp} = \frac{8V_s}{\pi} \sin \alpha \sin \frac{d}{2}$
 $\frac{2d}{N} = 45^\circ, 2d = 45 \times N = 45 \times 3$

$\Rightarrow d = \frac{45 \times 3}{2} = 67.5$

$\alpha = \frac{\pi - 2d}{N+1} + \frac{d}{N} = \frac{\pi - 135}{3+1} + \frac{67.5}{3} = 33.75$

$V_{olp} = \frac{8V_s}{\pi} \sin(33.75) \sin\left(\frac{67.5}{2}\right)$

$V_{olp} = 0.785 V_r$

3- ϕ Inverters:

* 3- ϕ Inverter: Supplies power to the 3- ϕ loads.
 Circuit diagram of 3- ϕ inverter is shown in figure.
 It consists of 6 thyristors and 6 feedback diodes.

* Feedback diodes allows current through them when load is reactive in nature (RL load).

* 3- ϕ inverter is also called as 6 step bridge inverter.

* For 1 cycle of 360°, each step has time interval of 60°.

* There are two possible modes of applying gating signals to the SCR's.

1. 180° mode of operation
2. 120° mode of operation

* In both modes gating pulses to the SCR's are applied and removed regularly at a time interval of 60°.

180°

* In this mode each SCR conducts for 180° time interval.
 * In this mode at any instant max. 3 SCR's are in conduction.

120°

* In this mode each SCR conducts for 120° time interval.
 * At any instant max. 2 SCR's are in conduction.

* In each leg SCR pairs (T₁, T₄, T₃, T₆, T₅, T₂) conduct with time interval of 180°.
 i.e. If SCR T₁ is fired at t=0, then SCR T₄ must be fired at t=180°.

* Upper group SCR's (T₁, T₃, T₅) conduct with a time interval of 120°. If T₁ is fired at t=0, T₃ must be fired at t=120° and T₅ at t=240°. Same is true for lower group SCR's (T₄, T₆, T₂).

Same point

Same point

$V_{ab} = \frac{2\sqrt{3}}{3} V_s$

$V_{bc} = \frac{2\sqrt{3}}{3} V_s$

$V_{ca} = \frac{2\sqrt{3}}{3} V_s$

180° mode:

Interval	Thyristor Conduct	Line to neutral Voltages			line to line Voltages		
		V _{ao}	V _{bo}	V _{co}	V _{ab}	V _{bc}	V _{ca}
I	T ₁ T ₅ T ₆	$\frac{V_s}{3}$	$-\frac{2V_s}{3}$	$-\frac{V_s}{3}$	V _s	-V _s	0
II	T ₁ T ₂ T ₆	$\frac{2V_s}{3}$	$-\frac{V_s}{3}$	$-\frac{V_s}{3}$	V _s	0	-V _s
III	T ₁ T ₂ T ₃	$\frac{V_s}{3}$	$\frac{V_s}{3}$	$-\frac{2V_s}{3}$	0	V _s	-V _s
IV	T ₂ T ₃ T ₄	$-\frac{V_s}{3}$	$-\frac{2V_s}{3}$	$-\frac{V_s}{3}$	-V _s	V _s	0
V	T ₃ T ₄ T ₅	$-\frac{2V_s}{3}$	$\frac{V_s}{3}$	$\frac{V_s}{3}$	-V _s	0	V _s
VI	T ₄ T ₅ T ₆	$-\frac{V_s}{3}$	$-\frac{V_s}{3}$	$\frac{2V_s}{3}$	0	-V _s	V _s

$V_{ab} = V_{ao} + V_{ob} = V_{ao} - V_{bo}$

$V_{bc} = V_{bo} + V_{oc} = V_{bo} - V_{co}$

$V_{ca} = V_{co} + V_{oa} = V_{co} - V_{ao}$

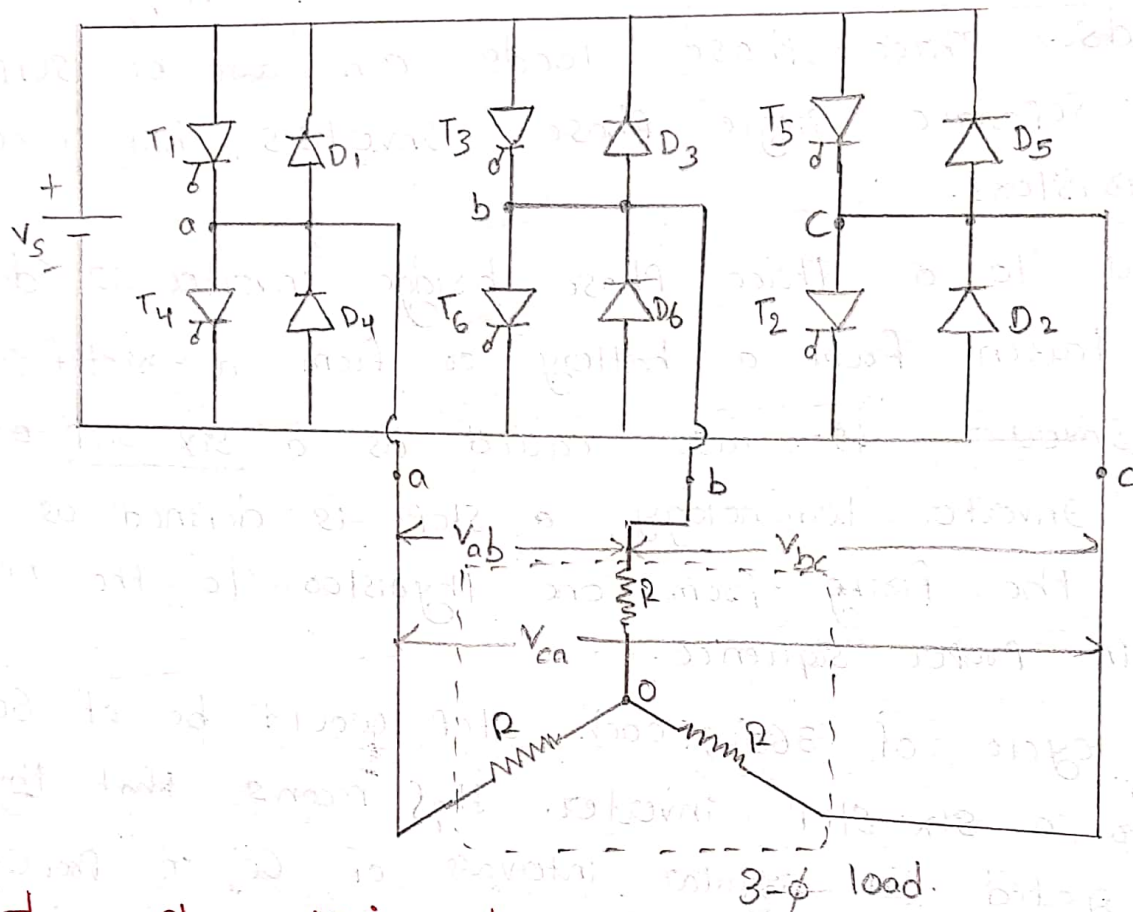


Fig 1
3- ϕ Inverter

i) Three Phase 180° mode VSI:

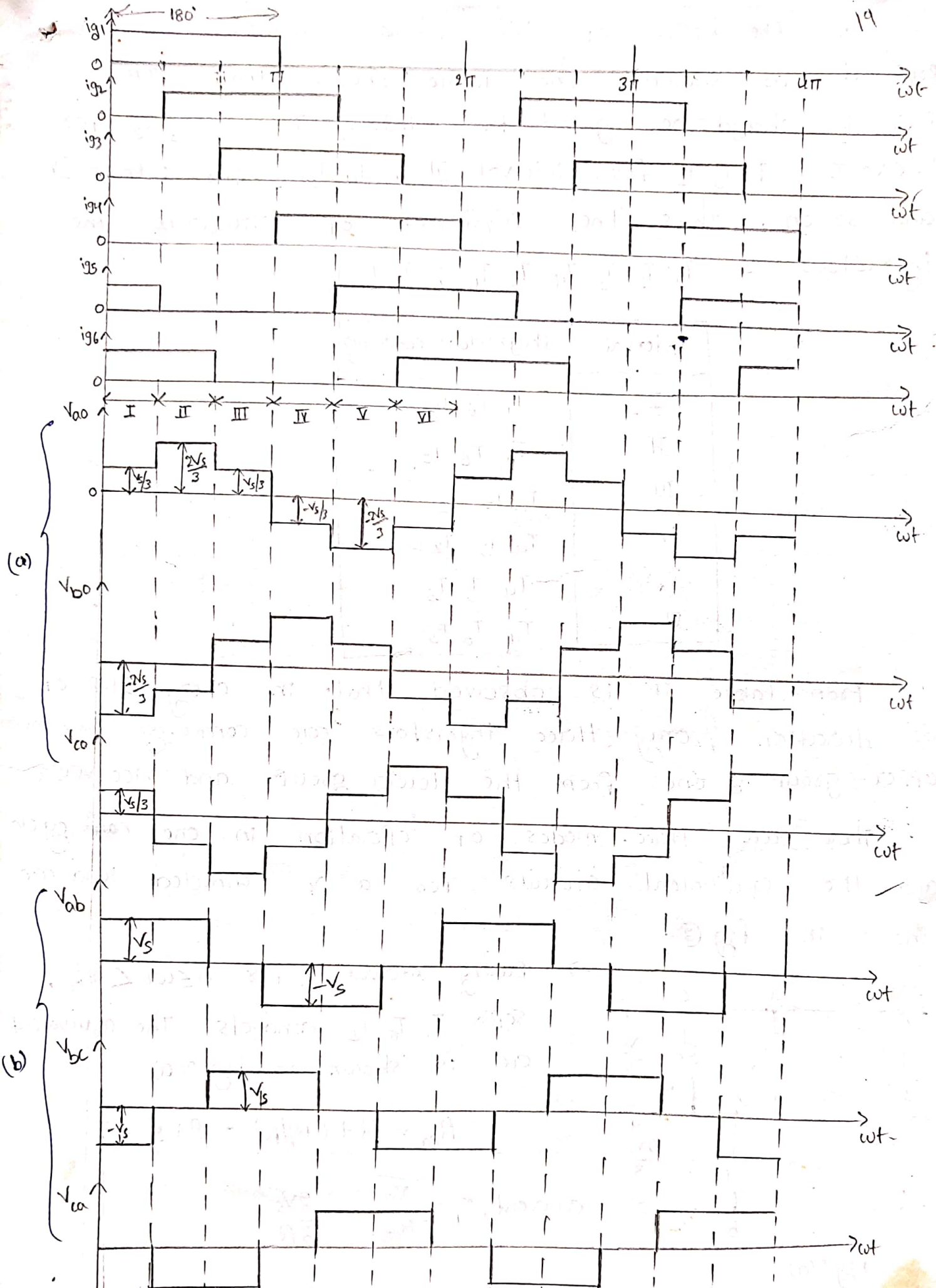
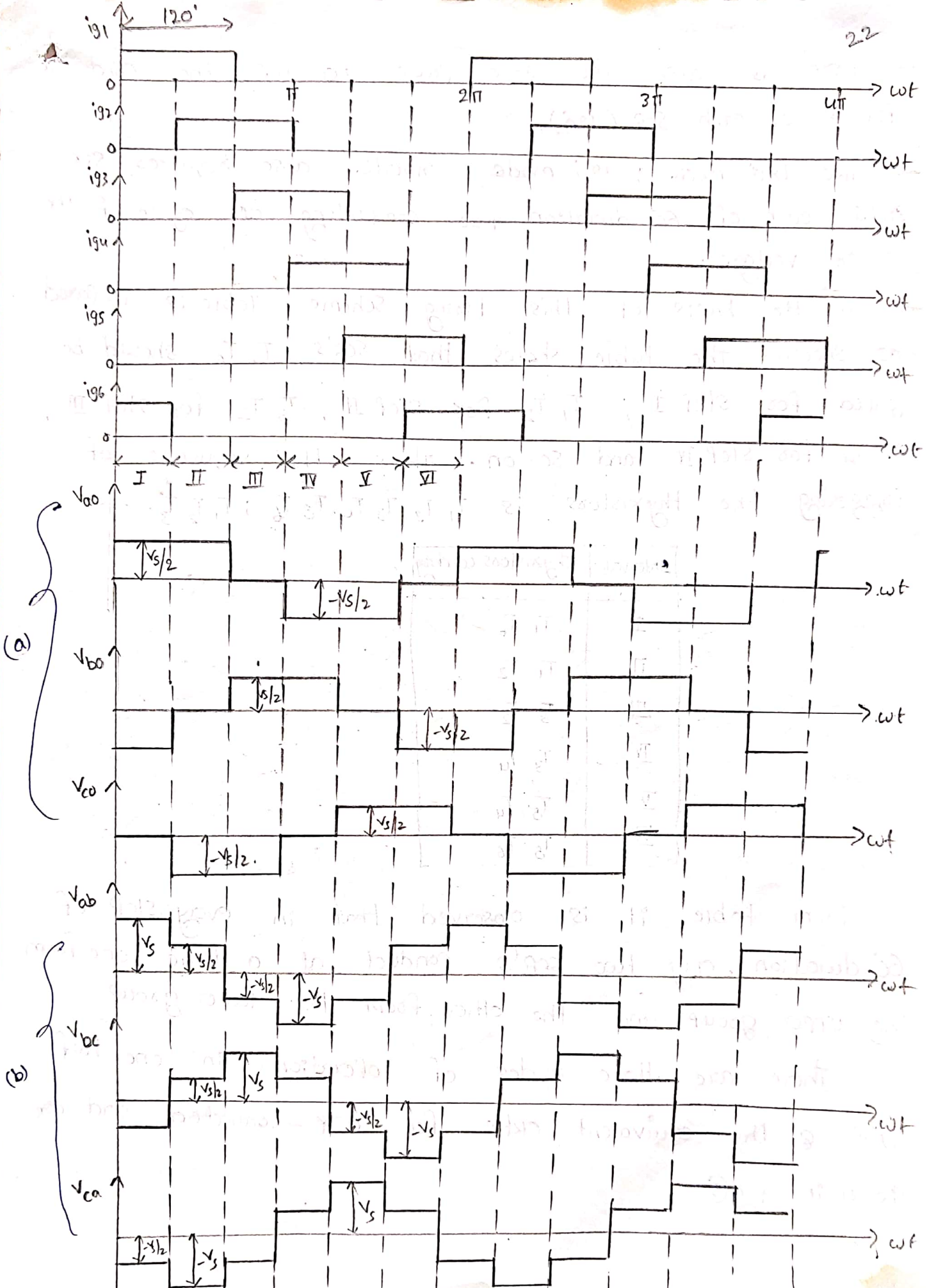


Fig (2): wave forms of voltage for 180° conduction.



fig(4): wfls of voltage for 120° conduction mode.

180° mode:

Interval	Thyristor Conduct	Line to neutral			line to line		
		V_{ao}	V_{bo}	V_{co}	V_{ab}	V_{bc}	V_{ca}
I	T_1, T_6	$\frac{V_s}{2}$	$-\frac{V_s}{2}$	0	V_s	$-\frac{V_s}{2}$	$-\frac{V_s}{2}$
II	T_1, T_2	$\frac{V_s}{2}$	0	$-\frac{V_s}{2}$	$\frac{V_s}{2}$	$\frac{V_s}{2}$	$-V_s$
III	T_2, T_3	0	$\frac{V_s}{2}$	$-\frac{V_s}{2}$	$-\frac{V_s}{2}$	$-V_s$	$-\frac{V_s}{2}$
IV	T_3, T_4	$-\frac{V_s}{2}$	$\frac{V_s}{2}$	0	$-V_s$	$\frac{V_s}{2}$	$\frac{V_s}{2}$
V	T_4, T_5	$-\frac{V_s}{2}$	0	$\frac{V_s}{2}$	$-\frac{V_s}{2}$	$-\frac{V_s}{2}$	V_s
VI	T_5, T_6	0	$-\frac{V_s}{2}$	$\frac{V_s}{2}$	$\frac{V_s}{2}$	$-V_s$	$\frac{V_s}{2}$

Important questions:

1. 120°, 180°
2. Cyclo Converter (step up, step down) (R-RL)
3. Buck regulator, boost regulator
4. AC Voltage controllers with RL load
5. Modes of operation of TRIAC
6. Step up chopper } Problems & theory for external
7. Step down chopper with RL and RLE load }
8. PWM techniques
9. 1- ϕ bridge inverter